A Model of Monetary Exchange in Over-the-Counter Markets*

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May 2014

Abstract

We develop a model of monetary exchange in over-the-counter markets to study the effects of monetary policy on asset prices and standard measures of financial liquidity, such as bid-ask spreads, trade volume, and the incentives of dealers to supply immediacy, both by participating in the market-making activity and by holding asset inventories on their own account. The theory predicts that asset prices carry a speculative premium that reflects the asset's marketability and depends on monetary policy as well as the market microstructure where it is traded. These liquidity considerations imply a positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous. The theory also exhibits rational expectations equilibria with recurring belief driven events that resemble liquidity crises, i.e., times of sharp persistent declines in asset prices, trade volume, and dealer participation in market-making activity, accompanied by large increases in spreads and abnormally long trading delays.

Keywords: Money; Liquidity; OTC markets; Asset prices; Fed model; Financial crises
JEL classification: D83, E31, E52, G12

*We thank the support from the C.V. Starr Center for Applied Economics at NYU.

1 Introduction

We develop a model of monetary exchange in financial over-the-counter (OTC) markets and use it to study some elementary questions in financial and monetary economics. Specifically, we consider a setting in which a financial asset that yields a dividend flow of consumption goods (e.g., an equity or a real bond) is traded by investors who have time-varying heterogeneous valuations for the dividend. In order to achieve the gains from trade that arise from their heterogeneous private valuations, investors participate in a bilateral market with random search that is intermediated by specialized dealers who have access to a competitive interdealer market. In the bilateral market, which has all the stylized features of a typical OTC market structure, investors and dealers seek to trade the financial asset using fiat money as a medium of exchange. Periodically, dealers and investors are also able to rebalance their portfolios in a frictionless (Walrasian) market.

We use the theory to study the role that the quantity of money plays in shaping asset prices in particular and the performance of OTC markets more generally. Since money serves as means of payment in financial transactions, the quantity of real balances affects the equilibrium allocation of the asset. Anticipated inflation reduces real balances and distorts the asset allocation by causing too many assets to remain in the hands of investors with relatively low valuations. We find that in a monetary equilibrium, the asset price is larger than the expected present discounted value that any agent assigns to the dividend stream. The difference between the transaction price and the highest individual valuation is a "speculative premium" that investors are willing to pay because they anticipate capital gains from reselling the asset to investors with higher valuations in the future. We show that the speculative premium and the asset price depend on the market structure where the asset is traded, e.g., both the premium and the asset price are decreasing in the effective bargaining power of dealers in the OTC market, as captured by the product of their trading probability and bargaining power in bilateral transactions with investors. Monetary policy also affects speculative motives and the resulting speculative premium. Anticipated inflation reduces the real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium and the real asset price are decreasing in the rate of (expected) inflation. This simple mechanism rationalizes the positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous and that, for lack of an alternative theory, has been attributed to money illusion since the 1970s. We also use the model to study the effects of monetary policy on standard measures of financial liquidity of OTC markets, such as the size of bid-ask spreads, the volume of trade, and the incentives of dealers to supply immediacy, both by choosing to participate in the market-making activity, as well as by holding asset inventories on their own account. We show that a version of the model in which dealer participation in market-making activities is endogenous exhibits rational expectations equilibria with recurring belief-driven events that resemble liquidity crises in which liquidity "dries up" and speculative premia "burst." Specifically, there are times when dealers withdraw from market making, which makes the assets that they intermediate difficult to trade, causing their prices and trade volume to fall abruptly at the same time that there is a sharp increase in the spreads and trading delays.

2 The model

Time is represented by a sequence of periods indexed by $t = 0, 1, ...$ Each time-period is divided into two subperiods where different activities take place. There is a continuum of infinitely lived agents called *investors*, each identified with a point in the set $\mathcal{I} = [0, 1]$. There is also a continuum of infinitely lived agents called *dealers*, each identified with a point in the set $\mathcal{D} = [0, v]$, where $v \in \mathbb{R}_+$. All agents discount payoffs across periods with the same factor, $\beta \in (0, 1)$. In every period there is a continuum of productive units (or *trees*) with measure $A^s \in \mathbb{R}_{++}$. Every productive tree yields an exogenous *dividend* $y_t \in \mathbb{R}_+$ of a perishable consumption good at the end of the first subperiod of period t. (Each tree yields the same dividend as every other tree, so y_t is also the aggregate dividend.) At the beginning of every period t, every tree is subject to an independent idiosyncratic shock that renders it permanently unproductive with probability $1 - \pi \in [0, 1)$ (unproductive trees physically disappear). If a tree remains productive, its dividend in period $t + 1$ is $y_{t+1} = \gamma_{t+1} y_t$ where γ_{t+1} is a nonnegative random variable with cumulative distribution function Γ, i.e., $\Pr(\gamma_{t+1} \leq \gamma) = \Gamma(\gamma)$, and mean $\bar{\gamma} \in (0, (\beta\pi)^{-1})$. The time-$t$ dividend becomes known to all agents at the beginning of period t, and at that time each tree that failed is replaced by a new tree that yields dividend y_t in the initial period and follows the same stochastic process as other productive trees thereafter (the dividend of the initial set of trees, $y_0 \in \mathbb{R}_{++}$, is given at $t = 0$). In the second subperiod of every period, every agent has access to a linear production technology that transforms a unit of the agent's effort into a unit of another kind of perishable homogeneous consumption good.

Each productive tree has outstanding one durable and perfectly divisible equity share that represents the bearer's ownership of the tree and confers him the right to collect the dividends. At the beginning of every period $t \geq 1$, each investor receives an endowment of $(1-\pi) A^s$ equity shares corresponding to the new trees created in that period. When a tree fails, its equity share disappears with the tree. There is a second financial instrument, money, which is intrinsically useless (it is not an argument of any utility or production function, and unlike equity, ownership of money does not constitute a right to collect any resources). The stock of money at time t is denoted A_t^m. The initial stock of money, $A_0^m \in \mathbb{R}_{++}$, is given, and $A_{t+1}^m = \mu A_t^m$, with $\mu \in \mathbb{R}_{++}$. A monetary authority injects or withdraws money via lump-sum transfers or taxes to investors in the second subperiod of every period. At the beginning of period $t = 0$, each investor is endowed with a portfolio of equity shares and money. All financial instruments are perfectly recognizable, cannot be forged, and can be traded among agents in every subperiod.

In the second subperiod of every period, all agents can trade the consumption good produced in that subperiod, equity shares, and money, in a spot Walrasian market. In the first subperiod of every period, trading is organized as follows: Investors and dealers can trade equity shares and money in a random bilateral *OTC market*, while dealers can also trade equity shares and money with other dealers in a spot Walrasian *interdealer market*. We use $\alpha \in [0,1]$ to denote the probability that an individual investor is able to contact another investor in the OTC market. Once the two investors have contacted each other, the pair negotiates a trade involving equity shares and money. We assume that, with probability $\eta \in [0,1]$, the terms of the trade are chosen by the investor who values the equity dividend the most, and by the other investor with complementary probability.[1] After the transaction has been completed, the investors part ways. Similarly, we use $\delta \in [0, \min(\upsilon, 1-\alpha)]$ to denote the probability that an individual investor is able to make contact with a dealer in the OTC market. The probability that a dealer contacts an investor is $\delta/\upsilon \equiv \kappa \in [0,1]$. Once a dealer and an investor have contacted each other, the pair negotiates the quantity of equity shares that the dealer will buy from, or sell to the investor in exchange for money. We assume that the terms of the trade between an investor and a dealer in the OTC market are chosen by the investor with probability $\theta \in [0,1]$, and by the dealer with probability $1-\theta$. After the transaction has been completed, the dealer and the investor part ways.[2] The timing assumption is that the round of OTC trade between investors and dealers

[1] In the event that both investors value the dividend the same, each gets selected to make a take-it-or-leave it offer with equal probability.

[2] See Zhang (2012) for an OTC model with long-term relationships between investors and dealers.

takes place in the first subperiod of a typical period t, and ends before trees yield dividends. Hence equity is traded *cum dividend* in the OTC market (and in the interdealer market) of the first subperiod, but *ex dividend* in the Walrasian market of the second subperiod. We assume that agents cannot make binding commitments, that there is no enforcement, and that histories of actions are private in a way that precludes any borrowing and lending, so any trade must be *quid pro quo.* This assumption and the structure of preferences described below create the need for a medium of exchange.[3]

An individual dealer's preferences are given by

$$\mathbb{E}_0^d \sum_{t=0}^{\infty} \beta^t (c_{td} - h_{td})$$

where c_{td} is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t, and h_{td} is the utility cost from exerting h_{td} units of effort to produce this good. The expectation operator $\mathbb{E}_0^d$ is with respect to the probability measure induced by the dividend process and the random trading process in the OTC market. Dealers get no utility from the dividend good.[4] An individual investor's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_{ti} y_{ti} + c_{ti} - h_{ti}\right)$$

where y_{ti} is the quantity of the dividend good that investor i consumes at the end of the first subperiod of period t, c_{ti} is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t, and h_{ti} is the utility cost from exerting h_{ti} units of effort to produce this good. The variable ε_{ti} denotes the realization of a preference shock that is distributed independently over time and across agents, with a differentiable cumulative distribution function G on the support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and $\bar{\varepsilon} = \int \varepsilon dG(\varepsilon)$. Investor i learns his realization ε_{ti} at the beginning of period t, before the OTC trading round. The expectation operator $\mathbb{E}_0$ is with respect to the probability measure induced by the dividend process, the investor's preference shock and the random trading process in the OTC market.

[3]Notice that under these conditions there cannot exist a futures market for fruit, so an agent who wishes to consume the fruit dividend must be holding the equity share at the time the dividend is paid. A similar assumption is typically made in search models of financial OTC trade, e.g., see Duffie et al. (2005) and Lagos and Rocheteau (2009).

[4]This assumption implies that dealers have no direct consumption motive for holding the equity share. It is easy to relax, but it is the standard benchmark in the search-based OTC literature, e.g., see Duffie et al. (2005) and Lagos and Rocheteau (2009), Lagos, Rocheteau and Weill (2011), and Weill (2007).

3 Efficiency

Consider a social planner who wishes to maximize the sum of all agents' expected discounted utilities, subject to the same meeting frictions that agents face in the decentralized formulation. Specifically, in the first subperiod of every period, the planner can only reallocate assets within the pairs of the measure α of investors who have contacted each other directly, and among all dealers and the measure δ of investors chosen at random from the rest of the population. Let $\mathcal{B}_t \subseteq \mathcal{I}$ denote the subset of investors who get a bilateral trading opportunity with another investor in the OTC market of period t. For any $i \in \mathcal{B}_t$, let $b(i) \in \mathcal{B}_t$ denote investor i's partner in the bilateral meeting. Notice that $\int_{\mathcal{B}_t} di = \alpha$ is the measure of investors who have an OTC meeting with another investor, and $\int_{\mathcal{B}_t} \mathbb{I}_{\{i \leq b(i)\}} di = \alpha/2$ is the total number of direct bilateral transactions between investors in the OTC market. We restrict attention to symmetric allocations (identical agents receive equal treatment). Let c_{tD} and h_{tD} denote a dealer's consumption and production of the homogeneous consumption good in the second subperiod of period t. Let $c_{tI}(\varepsilon)$ and $h_{tI}(\varepsilon)$ denote consumption and production of the homogeneous consumption good in the second subperiod of period t by an investor with idiosyncratic preference type ε. Let $\tilde{a}_{tD}$ denote the beginning-of-period-t (before depreciation) equity holding of a dealer, and let a'_{tD} denote the equity holding of a dealer at the end of the first subperiod of period t (after OTC trade). Let $\tilde{a}_{tI}$ denote the beginning-of-period-t (before depreciation and endowment) asset holding of an investor. Finally, let $\underline{a}_{tij}(\varepsilon_i, \varepsilon_j)$ denote the post-trade equity holding of an investor i with preference type ε_i who has a direct bilateral trade opportunity with an investor j with preference type ε_j, and let a'_{tI} denote a measure on $\mathcal{F}([\varepsilon_L, \varepsilon_H])$, the Borel σ-field defined on $[\varepsilon_L, \varepsilon_H]$. The measure a'_{tI} is interpreted as the distribution of post-trade asset holdings among investors with different preference types who contacted a dealer in the first subperiod of period t. With this notation, the planner's problem consists of choosing a nonnegative allocation,

$$\left\{ \tilde{a}_{tD}, a'_{tD}, c_{tD}, h_{tD}, \tilde{a}_{tI}, a'_{tI}, \left[\left(\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) \right)_{i \in \mathcal{B}_t}, c_{tI}(\varepsilon_i), h_{tI}(\varepsilon_i) \right]_{\varepsilon_i, \varepsilon_{b(i)} \in [\varepsilon_L, \varepsilon_H]} \right\}_{t=0}^{\infty}, \quad (1)$$

to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\delta \int_{[\varepsilon_L, \varepsilon_H]} \varepsilon y_t a'_{tI}(d\varepsilon) + \int_{\varepsilon_L}^{\varepsilon_H} \left[(1-\alpha-\delta) \varepsilon y_t a_{tI} + c_{tI}(\varepsilon) - h_{tI}(\varepsilon) \right] dG(\varepsilon) + v(c_{tD} - h_{tD}) \right.$$
$$\left. + \int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \leq b(i)\}} \left[\varepsilon_i \underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) + \varepsilon_{b(i)} \underline{a}_{tb(i)i}(\varepsilon_{b(i)}, \varepsilon_i) \right] y_t dG(\varepsilon_i) dG(\varepsilon_{b(i)}) di \right]$$

(the expectation operator $\mathbb{E}_0$ is with respect to the probability measure induced by the dividend process) subject to

$$\underline{a}_{tib(i)}\left(\varepsilon_i,\varepsilon_{b(i)}\right)+\underline{a}_{tb(i)i}\left(\varepsilon_{b(i)},\varepsilon_i\right)\leq 2a_{tI} \tag{2}$$

$$v\tilde{a}_{tD}+\tilde{a}_{tI}\leq A^s \tag{3}$$

$$va'_{tD}+\delta\int_{[\varepsilon_L,\varepsilon_H]}a'_{tI}\left(d\varepsilon\right)\leq va_{tD}+\delta a_{tI} \tag{4}$$

$$\int_{\varepsilon_L}^{\varepsilon_H}c_{tI}\left(\varepsilon\right)dG\left(\varepsilon\right)+vc_{tD}\leq\int_{\varepsilon_L}^{\varepsilon_H}h_{tI}\left(\varepsilon\right)dG\left(\varepsilon\right)+vh_{tD} \tag{5}$$

$$a_{tD}=\pi\tilde{a}_{tD} \tag{6}$$

$$a_{tI}=\pi\tilde{a}_{tI}+\left(1-\pi\right)A^s. \tag{7}$$

Proposition 1 *The efficient allocation satisfies the following three conditions for every t: (a) $\tilde{a}_{tD}=(A^s-\tilde{a}_{tI})/v=A^s/v$, (b) $\underline{a}_{tib(i)}\left(\varepsilon_i,\varepsilon_{b(i)}\right)=\mathbb{I}_{\{\varepsilon_{b(i)}<\varepsilon_i\}}2a_{tI}+\mathbb{I}_{\{\varepsilon_{b(i)}=\varepsilon_i\}}a^o$ for all $i\in\mathcal{B}_t$, with $a^o\in[0,2a_{tI}]$, and (c) $a'_{ti}\left(E\right)=\mathbb{I}_{\{\varepsilon_H\in E\}}\left[\pi/\delta+(1-\pi)\right]A^s$, where $\mathbb{I}_{\{\varepsilon_H\in E\}}$ is an indicator function that takes the value 1 if $\varepsilon_H\in E$, and 0 otherwise, for any $E\in\mathcal{F}\left([\varepsilon_L,\varepsilon_H]\right)$.*

According to Proposition 1, the efficient allocation is characterized by the following three properties: (a) only dealers carry equity between periods, (b) in bilateral direct trades between investors, all the equity shares are allocated to the highest valuation investor, and (c) among those investors who have a trading opportunity with a dealer, only those with the highest preference type hold equity shares at the end of the first subperiod.

4 Equilibrium

We begin with the formulation of the individual dealer's optimization problem during a typical period. Let $V_t^D\left(\boldsymbol{a}_t\right)$ denote the maximum expected discounted payoff of a dealer who enters the OTC round of period t with portfolio $\boldsymbol{a}_t\equiv(a_t^m,a_t^s)$. Let $W_t^D\left(\boldsymbol{a}_t\right)$ denote the maximum expected discounted payoff of a dealer who is holding portfolio $\boldsymbol{a}_t$ at the beginning of the second subperiod of period t. Let ϕ_t^m be the real price of money, and ϕ_t^s be the real *ex dividend* price of equity in the second subperiod of period t (both expressed in terms of the second-subperiod consumption good). Then,

$$
\begin{aligned}
W_t^D(\boldsymbol{a}_t) = \max_{c_t, h_t, \tilde{\boldsymbol{a}}_{t+1}} & \left[c_t - h_t + \beta \mathbb{E}_t V_{t+1}^D(\boldsymbol{a}_{t+1})\right] \\
\text{s.t. } c_t + \boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} & \leq h_t + \boldsymbol{\phi}_t \boldsymbol{a}_t, \\
c_t, h_t \in \mathbb{R}_+, & \ \tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2 \\
\boldsymbol{a}_{t+1} & = \left(\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s\right),
\end{aligned} \tag{8}
$$

where $\mathbb{E}_t$ is the conditional expectation over the next-period realization of the dividend, $\boldsymbol{\phi}_t \equiv (\phi_t^m, \phi_t^s)$, $\tilde{\boldsymbol{a}}_{t+1} \equiv \left(\tilde{a}_{t+1}^m, \tilde{a}_{t+1}^s\right)$, and $\boldsymbol{\phi}_t \boldsymbol{a}_t$ denotes the dot product of $\boldsymbol{\phi}_t$ and $\boldsymbol{a}_t$.

Let $\hat{W}_t^D(\boldsymbol{a}_t)$ denote the maximum expected discounted payoff of a dealer with portfolio $\boldsymbol{a}_t$ in the first subperiod of period t, conditional on not having contacted an investor in the OTC market. Since the unmatched dealer can still access the interdealer market,

$$
\begin{aligned}
\hat{W}_t^D(\boldsymbol{a}_t) = & \max_{\hat{a}_t^m, \hat{a}_t^s} W_t^D(\hat{a}_t^m, \hat{a}_t^s) \\
\text{s.t. } & \hat{a}_t^m + p_t \hat{a}_t^s \leq a_t^m + p_t a_t^s, \\
& \hat{a}_t^m, \hat{a}_t^s \in \mathbb{R}_+,
\end{aligned} \tag{9}
$$

where p_t is the dollar price of equity in the interdealer market of period t.

Next consider the situation of a dealer who enters the OTC round of trade of period t with portfolio $\boldsymbol{a}_{td}$, and contacts an investor with portfolio $\boldsymbol{a}_{ti}$ and preference type ε in the OTC market. With probability θ the terms of trade are determined by a take-it-or-leave-it offer by the investor, and the resulting post-trade portfolios of the investor and the dealer are denoted

$$
\begin{aligned}
&[\overline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \overline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] \\
&[\overline{a}_{d}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \overline{a}_{d}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)],
\end{aligned}
$$

respectively, where $\boldsymbol{\psi}_t \equiv (1/p_t, \boldsymbol{\phi}_t)$. With probability $1 - \theta$ the terms of trade are determined by a take-it-or-leave-it offer by the dealer, and the resulting post-trade portfolios of the investor and the dealer are

$$
\begin{aligned}
&[\overline{a}_{i}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \overline{a}_{i}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] \\
&[\overline{a}_{d^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \overline{a}_{d^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)],
\end{aligned}
$$

respectively.[5] We can now write the value function of a dealer who enters the OTC round of trade of period t with portfolio $\boldsymbol{a}_{td}$,

$$
\begin{aligned}
V_t^D(\boldsymbol{a}_{td}) = \kappa\theta \int \hat{W}_t^D\left[\overline{a}_d^m(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t),\overline{a}_d^s(\boldsymbol{a}_{ti},\boldsymbol{a}_{dt},\varepsilon;\boldsymbol{\psi}_t)\right] dH_t(\boldsymbol{a}_{ti},\varepsilon) \\
+ \kappa(1-\theta)\int \hat{W}_t^D\left[\overline{a}_{d^*}^m(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t),\overline{a}_{d^*}^s(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)\right] dH_t(\boldsymbol{a}_{ti},\varepsilon) \\
+ (1-\kappa)\hat{W}_t^D(\boldsymbol{a}_{td}), \qquad (10)
\end{aligned}
$$

where H_t is the joint cumulative distribution function over the preference types and portfolios held by the random investor whom the dealer may contact in the OTC market of period t.

We now analyze the investor's problem in a typical period. Let $V_t^I(\boldsymbol{a}_{ti},\varepsilon)$ denote the maximum expected discounted payoff of an investor who has preference type ε and is holding portfolio $\boldsymbol{a}_{ti} \equiv (a_{ti}^m, a_{ti}^s)$ at the beginning of the OTC round of period t. Let $W_t^I(\boldsymbol{a}_t)$ denote the maximum expected discounted payoff of an investor who is holding portfolio $\boldsymbol{a}_t$ at the beginning of the second subperiod of period t (after the trees have borne dividends). Then,

$$
\begin{aligned}
W_t^I(\boldsymbol{a}_t) = \max_{c_t,h_t,\tilde{\boldsymbol{a}}_{t+1}} \left[c_t - h_t + \beta\mathbb{E}_t \int V_{t+1}^I(\boldsymbol{a}_{t+1},\varepsilon')\, dG(\varepsilon')\right] \qquad (11) \\
\text{s.t. } c_t + \boldsymbol{\phi}_t\tilde{\boldsymbol{a}}_{t+1} \le h_t + \boldsymbol{\phi}_t\boldsymbol{a}_t + T_t \\
c_t, h_t \in \mathbb{R}_+,\ \tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2 \\
\boldsymbol{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi\tilde{a}_{t+1}^s + (1-\pi)A^s),
\end{aligned}
$$

where T_t is the real value of the time-t lump-sum monetary transfer (tax, if negative). Since ε is i.i.d. over time, $W_t^I(\boldsymbol{a}_t)$ is independent of ε and the portfolio that each investor chooses to carry into period $t+1$ is independent of ε. Consequently, in what follows we can write $dH_t(\boldsymbol{a}_{ti},\varepsilon) = dF_t^I(\boldsymbol{a}_{ti})\, dG(\varepsilon)$, where F_t^I is the joint cumulative distribution function of investors' money and equity holdings at the beginning of the OTC round of period-t.

Consider a bilateral meeting in the OTC trading round of period t, between an investor i with portfolio $\boldsymbol{a}_{ti}$ and preference type ε_i, and an investor j with portfolio $\boldsymbol{a}_{tj}$ and preference type ε_j. Let $\tilde{\eta}(\varepsilon_i,\varepsilon_j) \equiv \eta\mathbb{I}_{\{\varepsilon_j<\varepsilon_i\}} + (1-\eta)\mathbb{I}_{\{\varepsilon_i<\varepsilon_j\}} + (1/2)\mathbb{I}_{\{\varepsilon_i=\varepsilon_j\}}$ denote the probability that the investor with preference type ε_i has the power to make a take-it-or-leave-it offer in a bilateral trade with an investor with preference type ε_j. When investor i makes the take-it-or-leave-it

[5] In what follows, we will sometimes use $\overline{a}_{ti^*}^m$ to denote $\overline{a}_{i^*}^m(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)$, $\overline{a}_{td}^s$ to denote $\overline{a}_d^s(\boldsymbol{a}_{ti},\boldsymbol{a}_{ti},\varepsilon;\boldsymbol{\psi}_t)$, etc.

offer to investor j, the resulting post-trade portfolios of investors i and j are denoted

$$[\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]$$
$$[\underline{a}_j^m(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_j^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],$$

respectively. With probability $1 - \tilde{\eta}(\varepsilon_i, \varepsilon_j)$ the terms of trade are determined by a take-it-or-leave-it offer by investor j, and the resulting post-trade portfolios of investors i and j are

$$[\underline{a}_i^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_i^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]$$
$$[\underline{a}_{j^*}^m(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_{j^*}^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],$$

respectively.[6] We can now write the value function of an investor who enters the OTC round of period t with portfolio $\boldsymbol{a}_{it}$ and preference type ε_i,

$$\begin{aligned}
V_t^I(\boldsymbol{a}_{ti}, \varepsilon_i) = \delta \int & \theta \{\varepsilon_i y_t \overline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t) + \\
& W_t^I[\overline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t), \overline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t)]\} \, dF_t^D(\boldsymbol{a}_{td}) \\
& + \delta \int (1-\theta) \{\varepsilon_i y_t \overline{a}_i^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t) + \\
& W_t^I[\overline{a}_i^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t), \overline{a}_i^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t)]\} \, dF_t^D(\boldsymbol{a}_{td}) \\
& + \alpha \int \tilde{\eta}(\varepsilon_i, \varepsilon_j) \{\varepsilon_i y_t \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\
& W_t^I[\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i^*}^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]\} \, dH_t(\boldsymbol{a}_{tj}, \varepsilon_j) \\
& + \alpha \int [1 - \tilde{\eta}(\varepsilon_i, \varepsilon_j)] \{\varepsilon_i y_t \underline{a}_i^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\
& W_t^I[\underline{a}_i^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_i^s(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)]\} \, dH_t(\boldsymbol{a}_{tj}, \varepsilon_j) \\
& + (1 - \alpha - \delta) \left[\varepsilon_i y_t a_{ti}^s + W_t^I(\boldsymbol{a}_{ti})\right],
\end{aligned} \tag{12}$$

where F_t^D is the cumulative distribution function over portfolios held by the random dealer whom the dealer may contact in the OTC market of period t. Next, we characterize the outcomes of trades in the OTC and the interdealer markets.

The maximization problem (9) represents the portfolio problem of a dealer who did not contact an investor in the OTC market of period t. The solution is summarized as follows:

[6] In what follows, we will sometimes use $\underline{a}_{ti^*}^m$ to denote $\underline{a}_{i^*}^m(\boldsymbol{a}_{ti}, \boldsymbol{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)$, $\underline{a}_{tj}^s$ to denote $\underline{a}_j^s(\boldsymbol{a}_{tj}, \boldsymbol{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)$, etc.

Lemma 1 *A dealer who enters period t with portfolio $\boldsymbol{a}_t$ and does not contact an investor, enters the second subperiod with portfolio $(\hat{a}^m_{td}, \hat{a}^s_{td}) \equiv (\hat{a}^m_d(\boldsymbol{a}_t; \boldsymbol{\psi}_t), \hat{a}^s_d(\boldsymbol{a}_t; \boldsymbol{\psi}_t))$, given by*

$$\hat{a}^m_{td} = \begin{cases} = 0 & \text{if } p_t\phi^m_t < \phi^s_t \\ \in [0, a^m_t + p_t a^s_t] & \text{if } p_t\phi^m_t = \phi^s_t \\ = a^m_t + p_t a^s_t & \text{if } \phi^s_t < p_t\phi^m_t \end{cases} \quad \text{and} \quad \hat{a}^s_{td} = \begin{cases} a^s_t + \frac{1}{p_t} a^m_t & \text{if } p_t\phi^m_t < \phi^s_t \\ a^s_t + \frac{1}{p_t}(a^m_t - \hat{a}^m_{td}) & \text{if } p_t\phi^m_t = \phi^s_t \\ 0 & \text{if } \phi^s_t < p_t\phi^m_t, \end{cases}$$

and his maximum expected discounted payoff is

$$\hat{W}^D_t(\boldsymbol{a}_t) = \bar{\phi}_t (a^m_t + p_t a^s_t) + W^D_t(\boldsymbol{0}) \tag{13}$$

where $\bar{\phi}_t \equiv \max(\phi^m_t, \phi^s_t/p_t)$, and

$$\begin{aligned} W^D_t(\boldsymbol{0}) &= \max_{\tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}^2_+} \left[-\boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} + \beta \mathbb{E}_t V^D_{t+1}(\boldsymbol{a}_{t+1})\right] \\ \text{s.t. } \boldsymbol{a}_{t+1} &= (\tilde{a}^m_{t+1}, \pi \tilde{a}^s_{t+1}). \end{aligned} \tag{14}$$

If $p_t\phi^m_t < \phi^s_t$, then a dealer who holds any cash in the interdealer market can use a dollar to buy $1/p_t$ equity shares, and the net return from this trade equals ϕ^s_t/p_t (the real value of the equities in the Walrasian market of the second subperiod of period t) minus ϕ^m (the real cost of the trading strategy), which is strictly positive. Hence, it is optimal for the dealer to sell all his cash for equity if $p_t\phi^m_t < \phi^s_t$. Conversely, if $\phi^s_t < p_t\phi^m_t$, it is optimal for the dealer to sell any equity holdings he may have and carry only cash into the second subperiod of period t.

Consider the bargaining problem between an investor with preference type ε and portfolio (a^m_{ti}, a^s_{ti}) who contacts a dealer with portfolio (a^m_{td}, a^s_{td}) in the OTC market of period t. With probability θ the investor has the power to make a take-it-or-leave-it offer to the dealer. The investor chooses his offer of post-trade portfolios for himself, $(\overline{a}^m_{ti*}, \overline{a}^s_{ti*})$, and for the dealer, $(\overline{a}^m_{td}, \overline{a}^s_{td})$, by solving

$$\begin{aligned} &\max_{\overline{a}^m_{ti*}, \overline{a}^s_{ti*}, \overline{a}^m_{td}, \overline{a}^s_{td}} \left[\varepsilon y_t \overline{a}^s_{ti*} + W^I_t(\overline{a}^m_{ti*}, \overline{a}^s_{ti*})\right] \\ \text{s.t. } &\overline{a}^m_{ti*} + \overline{a}^m_{td} + p_t(\overline{a}^s_{ti*} + \overline{a}^s_{td}) \le a^m_{ti} + a^m_{td} + p_t(a^s_{ti} + a^s_{td}) \\ &\hat{W}^D_t(\overline{a}^m_{td}, \overline{a}^s_{td}) \ge \hat{W}^D_t(a^m_{td}, a^s_{td}) \\ &\overline{a}^m_{ti*}, \overline{a}^s_{ti*}, \overline{a}^m_{td}, \overline{a}^s_{td} \in \mathbb{R}_+. \end{aligned}$$

The first constraint requires that the combined value of the post-trade portfolios held by the investor and the dealer cannot exceed the combined value of their pre-trade portfolios. The second constraint is the dealer's individual rationality constraint.

With complementary probability $1-\theta$, the dealer has the power to make a take-it-or-leave-it offer to the investor. The dealer chooses his offer of post-trade portfolios for himself, $(\overline{a}^m_{td^*}, \overline{a}^s_{td^*})$, and for the dealer, $(\overline{a}^m_{ti}, \overline{a}^s_{ti})$, by solving

$$\max_{\overline{a}^m_{ti}, \overline{a}^s_{ti}, \overline{a}^m_{td^*}, \overline{a}^s_{td^*}} \hat{W}^D_t\left(\overline{a}^m_{td^*}, \overline{a}^s_{td^*}\right)$$

$$\begin{aligned}
\text{s.t. } & \overline{a}^m_{ti} + \overline{a}^m_{td^*} + p_t(\overline{a}^s_{ti} + \overline{a}^s_{td^*}) \leq a^m_{ti} + a^m_{td} + p_t(a^s_{ti} + a^s_{td}) \\
& \varepsilon y_t \overline{a}^s_{ti} + W^I_t\left(\overline{a}^m_{ti}, \overline{a}^s_{ti}\right) \geq \varepsilon y_t a^s_{ti} + W^I_t\left(a^m_{ti}, a^s_{ti}\right) \\
& \overline{a}^m_{ti}, \overline{a}^s_{ti}, \overline{a}^m_{td^*}, \overline{a}^s_{td^*} \in \mathbb{R}_+.
\end{aligned}$$

The first constraint requires that the combined value of the pre-trade portfolios is enough to finance the post-trade portfolios. The second constraint is the investor's individual rationality constraint. The following result summarizes the outcome of the bargaining game between an investor and a dealer.

Lemma 2 *Consider the bargaining problem between an investor i with portfolio (a^m_{ti}, a^s_{ti}) and preference type ε, and a dealer d with portfolio (a^m_{td}, a^s_{td}) in the OTC market of period t.*

(i) *With probability θ the investor chooses the terms of trade, and in this case the investor exits the meeting with post-trade portfolio $(\overline{a}^m_{ti^*}, \overline{a}^s_{ti^*})$ given by*

$$\overline{a}^m_{ti^*} \begin{cases} = 0 & \text{if } \varepsilon^*_t < \varepsilon \\ \in [0, a^m_{ti} + p_t a^s_{ti}] & \text{if } \varepsilon = \varepsilon^*_t \\ = a^m_{ti} + p_t a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases} \quad \text{and} \quad \overline{a}^s_{ti^*} = \begin{cases} a^s_{ti} + \frac{1}{p_t} a^m_{ti} & \text{if } \varepsilon^*_t < \varepsilon \\ a^s_{ti} + \frac{1}{p_t}\left(a^m_{ti} - \overline{a}^m_{ti^*}\right) & \text{if } \varepsilon = \varepsilon^*_t \\ 0 & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

where

$$\varepsilon^*_t \equiv \frac{p_t \phi^m_t - \phi^s_t}{y_t}. \tag{15}$$

The dealer's portfolio $(\overline{a}^m_{td}, \overline{a}^s_{td})$ that results from trading with the investor is given by

$$\overline{a}^m_{td} \in [0, a^m_{td} + p_t a^s_{td}] \quad \text{and} \quad \overline{a}^s_{td} = a^s_{td} + \frac{1}{p_t}\left(a^m_{td} - \overline{a}^m_{td}\right).$$

(ii) *With probability $1-\theta$ the dealer chooses the terms of trade, and in this case the investor exits the meeting with post-trade portfolio $(\overline{a}^m_{ti}, \overline{a}^s_{ti})$ given by*

$$\overline{a}^m_{ti} \begin{cases} = 0 & \text{if } \varepsilon^*_t < \varepsilon \\ \in [0, a^m_{ti} + p_t a^s_{ti}] & \text{if } \varepsilon = \varepsilon^*_t \\ = a^m_{ti} + p^o_t(\varepsilon) a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases} \quad \text{and} \quad \overline{a}^s_{ti} = \begin{cases} a^s_{ti} + \frac{1}{p^o_t(\varepsilon)} a^m_{ti} & \text{if } \varepsilon^*_t < \varepsilon \\ a^s_{ti} + \frac{1}{p_t}\left(a^m_{ti} - \overline{a}^m_{ti}\right) & \text{if } \varepsilon = \varepsilon^*_t \\ 0 & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$

where

$$p_t^o(\varepsilon) \equiv \left(\frac{\varepsilon y_t + \phi_t^s}{\varepsilon_t^* y_t + \phi_t^s}\right) p_t = \frac{\varepsilon y_t + \phi_t^s}{\phi_t^m}. \tag{16}$$

The dealer's portfolio $(\bar{a}_{td^*}^m, \bar{a}_{td^*}^s)$ *that results from trading with the investor is given by*

$$\bar{a}_{td^*}^m \begin{cases} \in [0, a_{td}^m + p_t a_{td}^s + [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)}] & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0, a_{td}^m + p_t a_{td}^s] & \text{if } \varepsilon = \varepsilon_t^* \\ \in [0, a_{td}^m + p_t a_{td}^s + [p_t - p_t^o(\varepsilon)] a_{ti}^s] & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

and

$$\bar{a}_{td^*}^s = \begin{cases} a_{td}^s + \frac{1}{p_t}[a_{td}^m + [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)} - \bar{a}_{td^*}^m] & \text{if } \varepsilon_t^* < \varepsilon \\ a_{td}^s + \frac{1}{p_t}(a_{td}^m - \bar{a}_{td^*}^m) & \text{if } \varepsilon = \varepsilon_t^* \\ a_{td}^s + \frac{1}{p_t}[a_{td}^m + [p_t - p_t^o(\varepsilon)] a_{ti}^s - \bar{a}_{td^*}^m] & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}$$

To interpret the results in Lemma 2, observe that (15) defines the preference type of the "marginal investor." That is, investors with preference type $\varepsilon < \varepsilon_t^*$ want to sell equity for cash, investors with preference type $\varepsilon > \varepsilon_t^*$ want to buy equity with cash, and the marginal investors with preference type $\varepsilon = \varepsilon_t^*$ are indifferent between buying or selling equity, as they have no gain from trading in the OTC market.[7] Consider an investor who has drawn preference type ε and meets a dealer in period t. If $p_t\phi_t^m < \varepsilon y_t + \phi_t^s$, or equivalently, if $\varepsilon_t^* < \varepsilon$, then the real value of a dollar to the investor is ϕ_t^m (the amount of second-subperiod goods he can buy in the following centralized market), which is smaller than $(\varepsilon y_t + \phi_t^s)/p_t$, namely the value to the investor of the (cum-dividend) equity position that can be purchased with a dollar in the interdealer market. Naturally, in this case the bargaining outcome is that the investor sells all his cash and uses it to purchase equity, regardless of whether the dealer or the investor has the bargaining power. Formally, in Lemma 2, $\bar{a}_{ti^*}^m = \bar{a}_{ti}^m = 0$ if $\varepsilon_t^* < \varepsilon$. Analogously, if $\varepsilon < \varepsilon_t^*$, then the investor sells all his equity for cash, both when he makes the offer, and when the dealer makes the offer (i.e., $\bar{a}_{ti^*}^s = \bar{a}_{ti}^s = 0$ if $\varepsilon < \varepsilon_t^*$ in the lemma). If $\varepsilon = \varepsilon_t^*$, the investor is indifferent between holding equity or cash; there are no gains from trade between him and the dealer.

The quantity of equity shares the investor gets for his cash holdings when $\varepsilon_t^* < \varepsilon$, and the amount of cash that he gets for his equity shares when $\varepsilon < \varepsilon_t^*$, however, do depend on whether the investor or the dealer has the bargaining power. If the investor has the bargaining power, he can effectively trade money for equity at the interdealer market price, p_t, i.e., he pays p_t dollars per share when he buys equity, and gets p_t dollars per share when he sells equity. Formally,

[7] Another way to interpret (15) is that given ε_t^*, $p_t\phi_t^m = \varepsilon_t^* y_t + \phi_t^s$ is the *cum dividend* real value of equity to the marginal investor in period t.

in Lemma 2, $\overline{a}^s_{ti^*} = a^s_{ti} + \frac{1}{p_t} a^m_{ti}$ if $\varepsilon^*_t < \varepsilon$, and $\overline{a}^m_{ti^*} = a^m_{ti} + p_t a^s_{ti}$ if $\varepsilon < \varepsilon^*_t$. If the dealer has the bargaining power, he offers less favorable terms of trade to the investor. Effectively, the bargaining outcome implies that the dealer lets the investor trade at $p^o_t(\varepsilon)$ dollars per share, rather than p_t dollars per share. Notice that $\partial p^o_t(\varepsilon)/\partial\varepsilon > 0$, so investors with higher preference types face a higher dollar price per share. Also, note that $p^o_t(\varepsilon) > p_t$ if and only if $\varepsilon^*_t < \varepsilon$. Thus the dealer charges $p^o_t(\varepsilon) > p_t$ dollars per share to an investor who wishes to buy equity (i.e., an investor with $\varepsilon^*_t < \varepsilon$), and pays $p^o_t(\varepsilon) < p_t$ dollars per share to an investor who wishes to sell equity (an investor with $\varepsilon < \varepsilon^*_t$). In other words, in a meeting where the dealer has the bargaining power, $p^o_t(\varepsilon)$ is the nominal *bid price* for investors who want to sell equity, or the nominal *ask price* for investors who want to buy equity. In terms of the lemma, this is why $\overline{a}^s_{ti} = a^s_{ti} + \frac{1}{p^o_t(\varepsilon)} a^m_{ti}$ if $\varepsilon^*_t < \varepsilon$, and $\overline{a}^m_{ti} = a^m_{ti} + p^o_t(\varepsilon) a^s_{ti}$ if $\varepsilon < \varepsilon^*_t$.

The indeterminacy in the dealer's portfolio follows from the fact that after having traded with the investor, the dealer can immediately retrade in the interdealer market, so all the dealer cares about is the value of his own *combined* post-trade portfolio. In fact, as the following corollary shows, the post-trade value of the dealer and the investor portfolios are uniquely pinned down.

Corollary 1 *Consider the bargaining problem between an investor i with portfolio (a^m_{ti}, a^s_{ti}) and preference type ε, and a dealer d with portfolio (a^m_{td}, a^s_{td}) in the OTC market of period t.*

(i) *With probability θ the investor chooses the terms of trade, and in this case the dollar value of the investor's and the dealer's post-trade portfolios are, respectively,*

$$\overline{a}^m_{ti^*} + p_t \overline{a}^s_{ti^*} = a^m_{ti} + p_t a^s_{ti}$$
$$\overline{a}^m_{td} + p_t \overline{a}^s_{td} = a^m_{td} + p_t a^s_{td}.$$

(ii) *With probability $1-\theta$ the dealer chooses the terms of trade, and in this case the dollar value of the investor's and the dealer's post-trade portfolios are, respectively,*

$$\overline{a}^m_{ti} + p_t \overline{a}^s_{ti} = \begin{cases} a^m_{ti} + p_t a^s_{ti} - \left[p^o_t(\varepsilon) - p_t\right] \frac{a^m_{ti}}{p^o_t(\varepsilon)} & \text{if } \varepsilon^*_t \leq \varepsilon \\ a^m_{ti} + p_t a^s_{ti} - \left[p_t - p^o_t(\varepsilon)\right] a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t \end{cases}$$
$$\overline{a}^m_{td^*} + p_t \overline{a}^s_{td^*} = \begin{cases} a^m_{td} + p_t a^s_{td} + \left[p^o_t(\varepsilon) - p_t\right] \frac{a^m_{ti}}{p^o_t(\varepsilon)} & \text{if } \varepsilon^*_t \leq \varepsilon \\ a^m_{td} + p_t a^s_{td} + \left[p_t - p^o_t(\varepsilon)\right] a^s_{ti} & \text{if } \varepsilon < \varepsilon^*_t. \end{cases}$$

Corollary 1 shows that the dealer extracts a transaction fee from the investor only when he has the bargaining power. For example, when the dealer encounters an investor with $\varepsilon > \varepsilon_t^*$ who wishes to purchase x shares, the dealer extracts $p_t^o(\varepsilon) - p_t = \frac{(\varepsilon - \varepsilon_t^*)y_t}{\varepsilon_t^* y_t + \phi_t^s} p_t$ dollars per share purchased by the investor, for a total fee of $\frac{(\varepsilon - \varepsilon_t^*)y_t}{\varepsilon_t^* y_t + \phi_t^s} p_t x$ dollars. In Lemma 2 and Corollary 1, $x = \bar{a}_{ti}^s - a_{ti}^s = \frac{1}{p_t^o(\varepsilon)} a_{ti}^m = \left(\frac{\varepsilon_t^* y_t + \phi_t^s}{\varepsilon y_t + \phi_t^s}\right) \frac{1}{p_t} a_{ti}^m$, so the total fee equals $\frac{(\varepsilon - \varepsilon_t^*)y_t}{\varepsilon y_t + \phi_t^s} a_{ti}^m$ dollars. Similarly, when the dealer encounters an investor with $\varepsilon < \varepsilon_t^*$ who wishes to sell a_{ti}^s shares, the dealer extracts $p_t - p_t^o(\varepsilon) = \frac{(\varepsilon_t^* - \varepsilon)y_t}{\varepsilon_t^* y_t + \phi_t^s} p_t$ dollars per share sold by the investor.

Consider a bilateral meeting in the OTC trading round of period t, between an investor i with portfolio a_{ti} and preference type ε_i, and an investor j with portfolio a_{tj} and preference type ε_j. With probability $\tilde{\eta}(\varepsilon_i, \varepsilon_j)$, investor i has the power to make a take-it-or-leave-it offer to investor j, and in that event investor i chooses an offer of post-trade portfolios for himself, $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$, and for investor j, $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$, by solving

$$\max_{\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s} \left[\varepsilon_i y_t \underline{a}_{ti^*}^s + W_t^I\left(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s\right)\right]$$

$$\begin{aligned} \text{s.t. } \underline{a}_{ti^*}^m + \underline{a}_{tj}^m &\leq a_{ti}^m + a_{tj}^m \\ \underline{a}_{ti^*}^s + \underline{a}_{tj}^s &\leq a_{ti}^s + a_{tj}^s \\ \varepsilon_j y_t \underline{a}_{tj}^s + W_t^I(\underline{a}_{tj}^m, \underline{a}_{tj}^s) &\geq \varepsilon_j y_t a_{tj}^s + W_t^I(a_{tj}^m, a_{tj}^s) \\ \underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s &\in \mathbb{R}_+. \end{aligned}$$

The first two constraints imply that in a bilateral meeting the two investors can (only) reallocate money and assets between themselves. The third constraint ensures that it is individually rational for investor j to accept i's offer. The following result summarizes the outcome of the bargaining game between two investors.

Lemma 3 *Consider the bargaining problem between an investor i with portfolio (a_{ti}^m, a_{ti}^s) and preference type ε_i, and an investor j with portfolio (a_{tj}^m, a_{tj}^s) and preference type ε_j in the OTC market of period t. Suppose that investor i has the power to choose the terms of trade, then his* *post-trade portfolio is* $\underline{\boldsymbol{a}}_{ti^*} = (\underline{a}^m_{ti^*}, \underline{a}^s_{ti^*})$ *with*

$$\underline{a}^s_{ti^*} \begin{cases} = a^s_{ti} + \min\left[\frac{a^m_{ti}}{p^o_t(\varepsilon_j)}, a^s_{tj}\right] & \text{if } \varepsilon_j < \varepsilon_i \\ \in \left[a^s_{ti} - \min\left[\frac{a^m_{tj}}{p^o_t(\varepsilon_j)}, a^s_{ti}\right], a^s_{ti} + \min\left[\frac{a^m_{ti}}{p^o_t(\varepsilon_j)}, a^s_{tj}\right]\right] & \text{if } \varepsilon_j = \varepsilon_i \\ = a^s_{ti} - \min\left[\frac{a^m_{tj}}{p^o_t(\varepsilon_j)}, a^s_{ti}\right] & \text{if } \varepsilon_i < \varepsilon_j \end{cases}$$

$$\underline{a}^m_{ti^*} = \begin{cases} a^m_{ti} - \min\left[p^o_t(\varepsilon_j) a^s_{tj}, a^m_{ti}\right] & \text{if } \varepsilon_j < \varepsilon_i \\ a^m_{ti} + p^o_t(\varepsilon_j)\left(a^s_{ti} - \underline{a}^s_{ti^*}\right) & \text{if } \varepsilon_j = \varepsilon_i \\ a^m_{ti} + \min\left[p^o_t(\varepsilon_j) a^s_{ti}, a^m_{tj}\right] & \text{if } \varepsilon_i < \varepsilon_j, \end{cases}$$

and investor j's post-trade portfolio is $\underline{\boldsymbol{a}}_{tj} = (\underline{a}^m_{tj}, \underline{a}^s_{tj})$*, with* $\underline{a}^s_{tj} = a^s_{tj} + a^s_{ti} - \underline{a}^s_{ti^*}$ *and* $\underline{a}^m_{tj} = a^m_{tj} + a^m_{ti} - \underline{a}^m_{ti^*}$*.*

In Lemma 3, if $\varepsilon_j < \varepsilon_i$, then investor i wishes to purchase all of investor j's equity. Since he has all the bargaining power, investor i sets the terms of trade at $p^o_t(\varepsilon_j)$ dollars per equity share, i.e., the dollar price of equity that makes investor j just indifferent between selling equity for dollars or not. The quantity of equity that investor i is able to purchase will depend on his money holdings, a^m_{ti}. If $a^m_{ti} \geq p^o_t(\varepsilon_j)a^s_{tj}$, then he buys all of investor j's equity holdings, a^s_{tj}, in exchange for $p^o_t(\varepsilon_j)a^s_{tj}$ dollars. If $a^m_{ti} < p^o_t(\varepsilon_j)a^s_{tj}$, then i gives investor j all his money holdings, a^m_{ti}, in exchange for $\frac{a^m_{ti}}{p^o_t(\varepsilon_j)}$ equity shares. Conversely, if $\varepsilon_i < \varepsilon_j$, then investor i wishes to sell all of his equity to investor j. Similarly, the quantity of equity that investor i will sell to investor j depends on j's money holdings, a^m_{tj}. If $a^m_{tj} \geq p^o_t(\varepsilon_j)a^s_{ti}$, then j buys all of investor i's equity holdings, a^s_{ti}, in exchange for $p^o_t(\varepsilon_j)a^s_{ti}$ dollars. If $a^m_{tj} < p^o_t(\varepsilon_j)a^s_{ti}$, then j gives investor i all his money holdings, a^m_{tj}, in exchange for $\frac{a^m_{tj}}{p^o_t(\varepsilon_j)}$ equity shares.

The bargaining outcomes can be substituted in the value functions (10) and (12) to obtain the following result.

Lemma 4 *Let* A^m_{It} *and* A^s_{It} *denote the quantity of money and shares held by all investors at the beginning of the OTC round of period t, respectively, i.e.,* $A^m_{It} = \int a^m_{ti} dF^I_t(\boldsymbol{a}_{ti})$ *and* $A^s_{It} = \int a^s_{ti} dF^I_t(\boldsymbol{a}_{ti})$*.*

(i) *The value function of a dealer who enters the OTC round of period t with portfolio* $\boldsymbol{a}_{td} = (a^m_{td}, a^s_{td})$ *is given by*

$$V^D_t(a^m_{td}, a^s_{td}) = \bar{\phi}_t(a^m_{td} + p_t a^s_{td}) + V^D_t(\mathbf{0}) \tag{17}$$

where

$$V_t^D(\mathbf{0}) \equiv \kappa(1-\theta)\bar{\phi}_t \left[A_{It}^m \int_{\varepsilon_t^*}^{\varepsilon_H} \frac{(\varepsilon-\varepsilon_t^*) y_t}{\varepsilon y_t + \phi_t^s} dG(\varepsilon) + p_t A_{It}^s \int_{\varepsilon_L}^{\varepsilon_t^*} \frac{(\varepsilon_t^*-\varepsilon) y_t}{\varepsilon_t^* y_t + \phi_t^s} dG(\varepsilon)\right] + W_t^D(\mathbf{0}).$$

(ii) *The value function of an investor who enters the OTC round of period t with portfolio $\boldsymbol{a}_{ti} = (a_{ti}^m, a_{ti}^s)$ and preference type ε_i is given by*

$$\begin{aligned} V_t^I(a_{ti}^m, a_{ti}^s, \varepsilon_i) = {}& \phi_t^m a_{ti}^m + (\varepsilon_i y_t + \phi_t^s) a_{ti}^s + W_t^I(\mathbf{0}) \\ & + \delta\theta \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}} \frac{(\varepsilon_i - \varepsilon_t^*) y_t}{\varepsilon_t^* y_t + \phi_t^s} \phi_t^m a_{ti}^m + \delta\theta \mathbb{I}_{\{\varepsilon_i < \varepsilon_t^*\}} (\varepsilon_t^* - \varepsilon_i) y_t a_{ti}^s \\ & + \alpha \int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \eta \frac{(\varepsilon_i - \varepsilon_j) y_t}{\varepsilon_j y_t + \phi_t^s} \min\left[\phi_t^m a_{ti}^m, (\varepsilon_j y_t + \phi_t^s) A_{It}^s\right] dG(\varepsilon_j) \\ & + \alpha \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} (1-\eta) \frac{(\varepsilon_j - \varepsilon_i) y_t}{\varepsilon_j y_t + \phi_t^s} \min\left[\phi_t^m A_{It}^m, (\varepsilon_j y_t + \phi_t^s) a_{ti}^s\right] dG(\varepsilon_j) \quad (18) \end{aligned}$$

where $\mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}}$ is an indicator function that takes the value 1 if $\varepsilon_t^* \leq \varepsilon_i$, and 0 otherwise.

Notice that the first term on the right side of $V_t^D(\mathbf{0})$ is the expected fee earned by a dealer in the OTC market of period t (the term inside the square bracket on the right side of $V_t^D(\mathbf{0})$ is the expected fee earned by a dealer when he makes an offer to an investor whose preference type, ε, is a random draw from G). To interpret (18), notice that the first line represents the value to the investor of holding the portfolio of money and equity until the end of the period. The remaining four terms represent the expected net gains from trading. For example, the factor that multiplies the indicator function $\mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}}$, i.e., $\delta\theta \frac{(\varepsilon_i - \varepsilon_t^*) y_t}{\varepsilon_t^* y_t + \phi_t^s} \phi_t^m a_{ti}^m$, is the expected net gain to the investor from exchanging money for shares in a trade with a dealer in the OTC market, and the factor that multiplies the indicator function $\mathbb{I}_{\{\varepsilon_i < \varepsilon_t^*\}}$, i.e., $\delta\theta(\varepsilon_t^* - \varepsilon_i) y_t a_{ti}^s$, is the expected net gain to the investor from exchanging shares for money in a trade with a dealer in the OTC market.[8] The last two terms on the right side of (18) represent the expected net

[8] Conditional on having drawn $\varepsilon_i \geq \varepsilon_t^*$, the investor contacts a dealer with probability δ and has the bargaining power with probability θ. In this case the investor sells a_{ti}^m dollars for $\frac{1}{p_t} a_{ti}^m$ shares, and his net payoff from this transaction is

$$(\varepsilon_i y_t + \phi_t^s) \frac{1}{p_t} a_{ti}^m - \phi_t^m a_{ti}^m = (\varepsilon_i y_t + \phi_t^s) \frac{\phi_t^m}{\varepsilon_t^* y_t + \phi_t^s} a_{ti}^m - \phi_t^m a_{ti}^m = \frac{(\varepsilon_i - \varepsilon_t^*) y_t}{\varepsilon_t^* y_t + \phi_t^s} \phi_t^m a_{ti}^m.$$

Conditional on having drawn $\varepsilon_i < \varepsilon_t^*$, the investor contacts a dealer with probability δ and has the bargaining power with probability θ. In this case the investor sells a_{ti}^s shares for p_t dollars each. The investor's net payoff from this transaction is $\phi_t^m p_t a_{ti}^s - (\varepsilon_i y_t + \phi_t^s) a_{ti}^s = (\varepsilon_t^* - \varepsilon_i) y_t a_{ti}^s$.

gains from trading with another investor in the OTC market.[9]

The following result uses Lemma 4 to characterize the solutions to the portfolio problems that a typical dealer and a typical investor solve in the second subperiod of period t.

Lemma 5 *Let* $\left(\tilde{a}^m_{t+1d}, \tilde{a}^s_{t+1d}\right)$ *and* $\left(\tilde{a}^m_{t+1i}, \tilde{a}^s_{t+1i}\right)$ *denote the portfolios chosen by a dealer and an investor, respectively, in the second subperiod of period* t*. The first-order necessary and sufficient conditions for optimization that these portfolios must satisfy are*

$$\phi^m_t \geq \beta \mathbb{E}_t \max\left(\phi^m_{t+1}, \phi^s_{t+1}/p_{t+1}\right) \tag{19}$$

$$\phi^s_t \geq \beta\pi \mathbb{E}_t \max\left(p_{t+1}\phi^m_{t+1}, \phi^s_{t+1}\right) \tag{20}$$

and

$$\begin{aligned}\phi^m_t \geq \beta \mathbb{E}_t \Bigg[& 1 + \delta\theta \int_{\varepsilon^*_{t+1}}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon^*_{t+1}) y_{t+1}}{\varepsilon^*_{t+1} y_{t+1} + \phi^s_{t+1}} dG(\varepsilon_i) \\ & + \alpha\eta \int_{\left[\frac{\phi^m_{t+1} a^m_{t+1i}}{A^s_{It+1}} - \phi^s_{t+1}\right]\frac{1}{y_{t+1}}}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j) y_{t+1}}{\varepsilon_j y_{t+1} + \phi^s_{t+1}} dG(\varepsilon_i)\, dG(\varepsilon_j) \Bigg] \phi^m_{t+1}\end{aligned} \tag{21}$$

$$\begin{aligned}\phi^s_t \geq \beta\pi \mathbb{E}_t \Bigg[& \phi^s_{t+1} + \Bigg(\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*_{t+1}} (\varepsilon^*_{t+1} - \varepsilon_i) dG(\varepsilon_i) \\ & + \alpha(1-\eta) \int_{\varepsilon_L}^{\left[\frac{\phi^m_{t+1} A^m_{It+1}}{a^s_{t+1i}} - \phi^s_{t+1}\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i)\, dG(\varepsilon_i)\, dG(\varepsilon_j) \Bigg) y_{t+1} \Bigg]\end{aligned} \tag{22}$$

where (19) holds with "=" if $\tilde{a}^m_{t+1d} > 0$*, (20) holds with "=" if* $\tilde{a}^s_{t+1d} > 0$*, (21) holds with "=" if* $\tilde{a}^m_{t+1i} > 0$*, and (22) holds with "=" if* $\tilde{a}^s_{t+1i} > 0$*.*

[9] Consider the penultimate term: with probability α the investor contacts another investor in the OTC market, if the other investor's preference type, ε_j, is smaller than ε_i, then the investor with preference type ε_i has the bargaining power with probability η and he spends $\min\left[a^m_{ti}, p^o_t(\varepsilon_j) A^s_{It}\right]$ dollars purchasing $\min\left[a^m_{ti}/p^o_t(\varepsilon_j), A^s_{It}\right]$ equity shares from the other investor, for a net gain from trade equal to

$$(\varepsilon_i y_t + \phi^s_t) \min\left[a^m_{ti}/p^o_t(\varepsilon_j), A^s_{It}\right] - \phi^m_t \min\left[a^m_{ti}, p^o_t(\varepsilon_j) A^s_{It}\right] = \frac{(\varepsilon_i - \varepsilon_j) y_t}{\varepsilon_j y_t + \phi^s_t} \min\left[\phi^m_t a^m_{ti}, (\varepsilon_j y_t + \phi^s_t) A^s_{It}\right].$$

Similarly, with probability α the investor contacts another investor in the OTC market, and if the other investor's preference type, ε_j, is larger than ε_i, then the investor with preference type ε_i has the bargaining power with probability $1-\eta$ and he sells $\min\left[A^m_{It}/p^o_t(\varepsilon_j), a^s_{ti}\right]$ equity shares in exchange for $\min\left[A^m_{It}, p^o_t(\varepsilon_j) a^s_{ti}\right]$ dollars, for a net gain from trade equal to

$$\phi^m_t \min\left[A^m_{It}, p^o_t(\varepsilon_j) a^s_{ti}\right] - (\varepsilon_i y_t + \phi^s_t) \min\left[A^m_{It}/p^o_t(\varepsilon_j), a^s_{ti}\right] = \frac{(\varepsilon_j - \varepsilon_i) y_t}{\varepsilon_j y_t + \phi^s_t} \min\left[\phi^m_t A^m_{It}, (\varepsilon_j y_t + \phi^s_t) a^s_{ti}\right].$$

Condition (19) is a dealer's Euler equation for money. The left side is the real cost of purchasing a dollar (in terms of the homogeneous good) in the second subperiod of period t. The right side is the discounted expected gain from this marginal dollar in the following period, i.e., the dealer can choose to hold on to the dollar until the second subperiod of $t+1$ to obtain ϕ_{t+1}^m homogeneous consumption goods, or he can sell the dollar in the interdealer market of the following OTC round for $1/p_{t+1}$ equity shares, each of which will be worth ϕ_{t+1}^s homogeneous goods in the second subperiod of $t+1$. Naturally, the dealer will choose the best of these two trading strategies. The dealer holds no money overnight if the left side of (19) exceeds the right side. Condition (20) is a dealer's Euler equation for equity shares. The left side is the real cost of purchasing a share (in terms of the homogeneous good) in the second subperiod of t. The right side is the discounted expected gain from this marginal share in the following period, i.e., the tree remains productive with probability π, and in that event the dealer can sell the share in the interdealer market of the following OTC round for p_{t+1} dollars, each of which will be worth ϕ_{t+1}^m homogeneous goods in the second subperiod of $t+1$, or he can choose to hold on to the share until the second subperiod of period $t+1$ to obtain ϕ_{t+1}^s homogeneous consumption goods. The dealer holds no equity overnight if the left side of (20) exceeds the right side.

Condition (21) is the investor's Euler equation for money. The left side is the real cost of purchasing a dollar in the second subperiod of period t. The right side is the discounted expected benefit from carrying this additional dollar into the following period, which consists of three components: (i) the expected benefit from holding the dollar until the second subperiod of period $t+1$ (i.e., if the investor does not spend the dollar in the OTC market), (ii) the expected gain from using the dollar to purchase equity from a dealer in the OTC market of period $t+1$, and (iii) the expected gain from using the dollar to purchase equity from another investor in the OTC market of period $t+1$. To interpret (21), it is useful to rewrite it as

$$\phi_t^m \geq \beta \mathbb{E}_t \left\{ \phi_{t+1}^m + \delta\theta \left[1 - G\left(\varepsilon_{t+1}^*\right)\right] \mathbb{E}\left[\frac{\varepsilon_i y_{t+1} + \phi_{t+1}^s}{p_{t+1}} - \phi_{t+1}^m \middle| \varepsilon_i \geq \varepsilon_{t+1}^* \right] \right.$$
$$\left. + \alpha\omega_{t+1}^m(a_{t+1i}^m, A_{It+1}^s)\eta \mathbb{E}\left[\frac{\varepsilon_i y_{t+1} + \phi_{t+1}^s}{p_{t+1}^o(\varepsilon_j)} - \phi_{t+1}^m \middle| (\varepsilon_i, \varepsilon_j) \in \Omega_{t+1}^m(a_{t+1i}^m, A_{It+1}^s) \right] \right\}$$

where $\mathbb{E}_t$ denotes the conditional expectation of y_{t+1}, $\mathbb{E}\left[\cdot|\cdot\right]$ denotes the conditional expectation of $(\varepsilon_i, \varepsilon_j)$, and for any $(a_{t+1i}^m, a_{t+1j}^s) \in \mathbb{R}_+^2$,

$$\Omega_{t+1}^m(a_{t+1i}^m, a_{t+1j}^s) = \left\{ (\varepsilon_i, \varepsilon_j) \in \left[\varepsilon_L, \varepsilon_H\right]^2 : \varepsilon_j < \varepsilon_i \text{ and } a_{t+1i}^m < p_{t+1}^o\left(\varepsilon_j\right) a_{t+1j}^s \right\}$$

and $\omega_{t+1}^{m}(a_{t+1i}^{m}, a_{t+1j}^{s}) \equiv \int\int \mathbb{I}_{\{(\varepsilon_i,\varepsilon_j)\in\Omega_{t+1}^{m}(a_{t+1i}^{m}, a_{t+1j}^{s})\}} dG(\varepsilon_i)\, dG(\varepsilon_j)$. With probability $\delta\theta[1 - G(\varepsilon_{t+1}^{*})]$, the investor contacts a dealer in the OTC market, has bargaining power, and wishes to purchase equity. In this event, he uses the marginal dollar worth ϕ_{t+1}^{m} to purchase $\frac{1}{p_{t+1}}$ equity shares each of which yields expected utility from the dividend equal to $\mathbb{E}\left[\varepsilon_i | \varepsilon_i \geq \varepsilon_{t+1}^{*}\right] y_{t+1}$, and a resale value of ϕ_{t+1}^{s} homogeneous goods in the second subperiod of $t+1$. With probability α, investor, call him i, contacts another investor, e.g., investor j. Then $\omega_{t+1}^{m}(a_{t+1i}^{m}, A_{It+1}^{s})\eta$ denotes the joint probability that i's preference type is higher than j's (so i acts as a buyer of equity), and i has bargaining power (which happens with conditional probability η), and the bilateral gains from trade are constrained by i's money holdings (which given i's money holdings, a_{t+1i}^{m}, and j's equity holdings, A_{It+1}^{s}, at the time of the trade, occurs if the bilateral dollar price of equity is large enough, i.e., if j's individual valuation of equity, ε_j, is large enough). In this event, carrying an additional dollar into period $t+1$ helps investor i reap gains from trade in the bilateral trade with the other investor, and i's expected gain from trading the marginal dollar is the (conditional expected) value of the additional equity he purchases, i.e., $\frac{1}{p_{t+1}^{o}(\varepsilon_j)}$ equity shares each worth $\varepsilon_i y_{t+1} + \phi_{t+1}^{s}$, minus the value of the dollar, ϕ_{t+1}^{m}.

Condition (22) is the investor's Euler equation for equity. To interpret this condition it is useful to rewrite it as

$$\begin{aligned}\phi_t^{s} \geq \beta\pi\mathbb{E}_t\Big\{&\bar{\varepsilon}y_{t+1} + \phi_{t+1}^{s} + \delta\theta G\left(\varepsilon_{t+1}^{*}\right)\mathbb{E}\left[p_{t+1}\phi_{t+1}^{m} - \left(\varepsilon_i y_{t+1} + \phi_{t+1}^{s}\right)|\varepsilon_i \leq \varepsilon_{t+1}^{*}\right] \\ &+ \alpha\omega_{t+1}^{s}(a_{t+1i}^{s}, A_{It+1}^{m})(1-\eta)\,\mathbb{E}\left[p_{t+1}^{o}\left(\varepsilon_j\right)\phi_{t+1}^{m} - \left(\varepsilon_i y_{t+1} + \phi_{t+1}^{s}\right)|(\varepsilon_i,\varepsilon_j) \in \Omega_{t+1}^{s}(a_{t+1i}^{s}, A_{It+1}^{m})\right]\Big\}\end{aligned}$$

where for any $(a_{t+1i}^{s}, a_{t+1j}^{m}) \in \mathbb{R}_{+}^{2}$,

$$\Omega_{t+1}^{s}(a_{t+1i}^{s}, a_{t+1j}^{m}) = \left\{(\varepsilon_i,\varepsilon_j) \in [\varepsilon_L, \varepsilon_H]^2 : \varepsilon_i < \varepsilon_j \text{ and } p_{t+1}^{o}\left(\varepsilon_j\right) a_{t+1i}^{s} < a_{t+1j}^{m}\right\}$$

and $\omega_{t+1}^{s}(a_{t+1i}^{s}, a_{t+1j}^{m}) \equiv \int\int \mathbb{I}_{\{(\varepsilon_i,\varepsilon_j)\in\Omega_{t+1}^{s}(a_{t+1i}^{s}, a_{t+1j}^{m})\}} dG(\varepsilon_i)\, dG(\varepsilon_j)$. The left side is the real cost of purchasing an additional equity share in the second subperiod of t. The right side is the discounted expected benefit from carrying an additional equity share into the following period, which consists of three terms. First, $\bar{\varepsilon}y_{t+1} + \phi_{t+1}^{s}$, the expected benefit of holding the equity share until the end of period $t+1$ (i.e., if the investor does not sell the equity in the OTC market). Second, with probability $\delta\theta G(\varepsilon_{t+1}^{*})$, the investor contacts a dealer in the OTC market, has bargaining power, and wishes to sell equity. In this event, he obtains $p_{t+1}\phi_{t+1}^{m}$ dollars for selling the marginal equity share which he expects to value $\mathbb{E}\left[\varepsilon_i y_{t+1} + \phi_{t+1}^{s} | \varepsilon_i \leq \varepsilon_{t+1}^{*}\right]$. Finally, with probability α investor i contacts another investor j in the OTC market. Then

$\omega^s_{t+1}(a^s_{t+1i}, A^m_{It+1})(1-\eta)$ denotes the joint probability that i's preference type is lower than j's (so i acts as a seller of equity), and i has bargaining power (which happens with conditional probability $1-\eta$), and the bilateral gains from trade are constrained by i's equity holdings (which given i's equity holdings, a^s_{t+1i}, and j's money holdings, A^m_{It+1}, at the time of the trade, occurs if the bilateral dollar price of equity is low enough, i.e., if j's individual valuation of equity, ε_j, is low enough). In this event, an additional equity share helps investor i reap gains from trade in the bilateral trade with the other investor, and i's expected gain from trading the marginal share is the (conditional expected) value of the real balances he receives, i.e., $p^o_{t+1}(\varepsilon_j)\,\phi^m_{t+1}$, minus the (conditional expected) value of the equity share he sells, i.e., $\varepsilon_i y_{t+1} + \phi^s_{t+1}$.

Let A^m_{Dt+1} and A^s_{Dt+1} denote the quantities of money and equity shares, respectively, held by all dealers at the beginning of the OTC round of period $t+1$, i.e., $A^m_{Dt+1} = v \int a^m_{t+1d} dF^D_{t+1}(\boldsymbol{a}_{t+1d})$, and $A^s_{Dt+1} = v \int a^s_{t+1d} dF^D_{t+1}(\boldsymbol{a}_{t+1d})$. Let $\tilde{A}^m_{Dt+1}$ and $\tilde{A}^s_{Dt+1}$ denote the total quantities of money and shares held by all dealers at the end of period t, i.e., $A^m_{Dt+1} = \tilde{A}^m_{Dt+1}$ and $A^s_{Dt+1} = \pi \tilde{A}^s_{Dt+1}$. Similarly, let $\tilde{A}^m_{It+1}$ and $\tilde{A}^s_{It+1}$ denote the total quantities of money and shares held by all investors at the end of period t, i.e., $A^m_{It+1} = \tilde{A}^m_{It+1}$ and $A^s_{It+1} = \pi \tilde{A}^s_{It+1} + (1-\pi) A^s$. Let $\bar{A}^m_{Dt}$ and $\bar{A}^s_{Dt}$ denote the quantity of money and shares held after the OTC round of trade of period t by all the dealers who are able to trade in the first subperiod. Similarly, let $\bar{A}^m_{It}$ and $\bar{A}^s_{It}$ denote the quantity of money and shares held after the OTC round of trade of period t by all the investors who are able to trade in the first subperiod. For $k = s, m$,

$$\begin{aligned}\bar{A}^k_{Dt} = {} & \kappa v \theta \int \hat{a}^k_d \left[\bar{\boldsymbol{a}}_d(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t\right] dF^D_t(\boldsymbol{a}_{td})\, dF^I_t(\boldsymbol{a}_{ti})\, dG(\varepsilon) \\ & + \kappa v (1-\theta) \int \hat{a}^k_d \left[\bar{\boldsymbol{a}}_{d^*}(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t\right] dF^D_t(\boldsymbol{a}_{td})\, dF^I_t(\boldsymbol{a}_{ti})\, dG(\varepsilon) \\ & + (1-\kappa) v \int \hat{a}^k_d(\boldsymbol{a}_{td}; \boldsymbol{\psi}_t) dF^D_t(\boldsymbol{a}_{td})\end{aligned}$$

and

$$\bar{A}^k_{It} = \delta \int \left[\theta \bar{a}^k_{i^*}(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) + (1-\theta)\, \bar{a}^k_i(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)\right] dF^D_t(\boldsymbol{a}_{td})\, dF^I_t(\boldsymbol{a}_{ti})\, dG(\varepsilon)$$

where $\bar{\boldsymbol{a}}_d(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) \equiv (\bar{a}^m_d(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}^s_d(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t))$. We are now ready to define equilibrium.

Definition 1 *An equilibrium is a sequence of terms of trade in the OTC market*

$$\{\langle \bar{\boldsymbol{a}}_{td}, \bar{\boldsymbol{a}}_{td^*}, \hat{\boldsymbol{a}}_{td}\rangle_{d\in\mathcal{D}}, \langle \bar{\boldsymbol{a}}_{ti^*}, \bar{\boldsymbol{a}}_{ti}, \underline{\boldsymbol{a}}_{ti^*}, \underline{\boldsymbol{a}}_{ti}\rangle_{i\in\mathcal{I}}\}_{t=0}^{\infty},$$

as given in Lemma 1, Lemma 2, and Lemma 3, together with a sequence of asset holdings

$$\{\langle \boldsymbol{a}_{t+1d}, \tilde{\boldsymbol{a}}_{t+1d}\rangle_{d\in\mathcal{D}}, \langle \boldsymbol{a}_{t+1i}, \tilde{\boldsymbol{a}}_{t+1i}\rangle_{i\in\mathcal{I}}\}_{t=0}^{\infty}$$

and prices $\{\boldsymbol{\psi}_t\}_{t=0}^{\infty} \equiv \{1/p_t, \phi_t^m, \phi_t^s\}_{t=0}^{\infty}$*, such that for all* t*, (i) the asset allocation solves the individual optimization problems (8) and (11) taking prices as given, and (ii) prices are such that all Walrasian markets clear, i.e.,* $\tilde{A}_{Dt+1}^s + \tilde{A}_{It+1}^s = A^s$ *(the end-of-period-t Walrasian market for equity),* $\tilde{A}_{Dt+1}^m + \tilde{A}_{It+1}^m = A_{t+1}^m$ *(the end-of-period-t Walrasian market for money), and* $\bar{A}_{Dt}^k + \bar{A}_{It}^k = A_{Dt}^k + \delta A_{It}^k$ *for* $k = s, m$ *(the period-t OTC interdealer market for equity and money). An equilibrium is "monetary" if* $\phi_t^m > 0$ *for all* t*, and "nonmonetary" otherwise.*

In what follows, we specialize the analysis to stationary equilibria. That is, equilibria in which asset holdings are constant over time, i.e., $A_{Dt}^s = A_D^s$ and $A_{It}^s = A_I^s$, real asset prices are time-invariant functions of the aggregate dividend, i.e., $\phi_t^s = \phi^s y_t$, $p_t \phi_t^m \equiv \bar{\phi}_t^s = \bar{\phi}^s y_t$, $\phi_t^m A_{It}^m = Z y_t$, and $\phi_t^m A_{Dt}^m = Z_D y_t$. Hence, in a stationary equilibrium, $\varepsilon_t^* = \bar{\phi}^s - \phi^s \equiv \varepsilon^*$, $\phi_{t+1}^s/\phi_t^s = \bar{\phi}_{t+1}^s/\bar{\phi}_t^s = \gamma_{t+1}$, $\phi_t^m/\phi_{t+1}^m = \mu/\gamma_{t+1}$, and $p_{t+1}/p_t = \mu$. Throughout the analysis we let $\bar{\beta} \equiv \beta\bar{\gamma}$ and maintain the assumption $\mu > \bar{\beta}$, but the following proposition considers the limiting case $\mu \to \bar{\beta}$.

Proposition 2 *The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as* $\mu \to \bar{\beta}$*.*

Let $q_{t,k}^B$ denote the nominal price in the second subperiod of period t of an N-period risk-free pure-discount nominal bond that matures in period $t+k$, for $k = 0, 1, 2, ..., N$ (so k is the number of periods until the bond matures). Assume that the bond is illiquid in the sense that it cannot be traded in the OTC market. Then in a stationary monetary equilibrium, $q_{t,k}^B = (\bar{\beta}/\mu)^k$, and

$$\iota = \frac{\mu - \bar{\beta}}{\bar{\beta}} \tag{23}$$

is the time-t nominal yield to maturity of the bond with k periods until maturity.[10] Thus, the optimal monetary policy described in Proposition 2 in which the money supply grows at rate $\bar{\beta}$ can be interpreted as a policy that implements the *Friedman rule*, i.e., $\iota = 0$ for all contingencies at all dates.

[10]See Lemma 9 in the appendix for details.

4.1 Pure-dealer OTC market

In this section we consider the case with $\alpha = 0$, i.e., a market in which all OTC trade is intermediated by dealers and there is no direct bilateral trade among investors. For the analysis that follows, it is convenient to define

$$\hat{\mu} \equiv \bar{\beta}\left[1 + \frac{(1-\delta\theta)\left(1-\bar{\beta}\pi\right)(\hat{\varepsilon}-\bar{\varepsilon})}{\hat{\varepsilon}}\right] \quad \text{and} \quad \bar{\mu} \equiv \bar{\beta}\left[1 + \frac{\delta\theta\left(1-\bar{\beta}\pi\right)(\bar{\varepsilon}-\varepsilon_L)}{\bar{\beta}\pi\bar{\varepsilon} + \left(1-\bar{\beta}\pi\right)\varepsilon_L}\right] \tag{24}$$

where $\hat{\varepsilon} \in [\bar{\varepsilon}, \varepsilon_H]$ is the unique solution to

$$\bar{\varepsilon} - \hat{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon)\, d\varepsilon = 0. \tag{25}$$

Lemma 6 (in the appendix) establishes that $\hat{\mu} < \bar{\mu}$. The following proposition summarizes the equilibrium set.

Proposition 3 *Assume $\alpha = 0$. (i) A nonmonetary equilibrium exists for any parametrization. (ii) There is no stationary monetary equilibrium if $\mu \geq \bar{\mu}$. (iii) In the nonmonetary equilibrium, $A_I^s = A^s - A_D^s = A^s$ (only investors hold equity shares), there is no trade in the OTC market, and the equity price in the Walrasian market is*

$$\phi_t^s = \phi^s y_t, \text{ with } \phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}. \tag{26}$$

(iv) If $\mu \in (\bar{\beta}, \bar{\mu})$, then there is one stationary monetary equilibrium; asset holdings of dealers and investors at the beginning of the OTC round of period t are $A_{Dt}^m = A_t^m - A_{It}^m = 0$ and

$$A_D^s = A^s - A_I^s \begin{cases} = \pi A^s & \text{if } \bar{\beta} < \mu < \hat{\mu} \\ \in [0, \pi A^s] & \text{if } \mu = \hat{\mu} \\ = 0 & \text{if } \hat{\mu} < \mu < \bar{\mu}, \end{cases}$$

and asset prices are

$$\phi_t^s = \phi^s y_t, \text{ with } \phi^s = \begin{cases} \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\varepsilon^* & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon\right] & \text{if } \hat{\mu} < \mu < \bar{\mu} \end{cases} \tag{27}$$

$$\bar{\phi}_t^s = \bar{\phi}^s y_t, \text{ with } \bar{\phi}^s = \varepsilon^* + \phi^s \tag{28}$$

$$\phi_t^m = Z \frac{y_t}{A_t^m} \tag{29}$$

$$p_t = \frac{\bar{\phi}^s}{Z} A_t^m \tag{30}$$

where

$$Z = \frac{A_D^s + \delta G(\varepsilon^*) A_I^s}{\delta\theta \left[1 - G(\varepsilon^*)\right] \frac{1}{\varepsilon^* + \phi^s} + \delta \left(1 - \theta\right) \int_{\varepsilon^*}^{\varepsilon_H} \frac{1}{\varepsilon + \phi^s} dG\left(\varepsilon\right)}, \tag{31}$$

and for any $\mu \in \left(\bar{\beta}, \bar{\mu}\right)$, $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\frac{\left(1 - \bar{\beta}\pi\right) \int_{\varepsilon^*}^{\varepsilon_H} \left[1 - G\left(\varepsilon\right)\right] d\varepsilon}{\varepsilon^* + \bar{\beta}\pi \left[\bar{\varepsilon} - \varepsilon^* + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G\left(\varepsilon\right) d\varepsilon\right] \mathbb{I}_{\{\hat{\mu} < \mu\}}} - \frac{\mu - \bar{\beta}}{\bar{\beta}\delta\theta} = 0. \tag{32}$$

(v) (a) As $\mu \to \bar{\mu}$, $\varepsilon^* \to \varepsilon_L$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi} \bar{\varepsilon} y_t$*. (b) As* $\mu \to \bar{\beta}$, $\varepsilon^* \to \varepsilon_H$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi} \varepsilon_H y_t$*.*

In the nonmonetary equilibrium, dealers are inactive and equity shares are held only by investors. With no valued money, investors and dealers cannot exploit the gains from trade that arise from the heterogeneity in preference types in the first subperiod of every period, and the equilibrium real asset price, $\phi^s = \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi} \bar{\varepsilon} y$, is equal to the expected discounted value of the dividend stream since the equity share is not traded. (Shares can be traded in the Walrasian market of the second subperiod, but gains from trade at that stage are nil.) The stationary monetary equilibrium exists only if the inflation rate is not too high, i.e., if $\mu < \bar{\mu}$. In the monetary equilibrium, the marginal preference type, ε^*, which according to Lemma 2 partitions the set of investors into those who buy and those who sell the asset when they meet a dealer in the OTC market, is characterized in part (*iv*) of Proposition 3. In the OTC market, investors with $\varepsilon < \varepsilon^*$ who contact dealers sell all their equity holdings for money, while investors with $\varepsilon > \varepsilon^*$ who contact dealers spend all their money buying equity. Thus unlike what happens in the nonmonetary equilibrium, the OTC market is active in the monetary equilibrium, and it is easy to show that the marginal type, ε^*, is strictly decreasing in the rate of inflation, i.e., $\frac{\partial \varepsilon^*}{\partial \mu} < 0$ both for $\mu \in \left(\bar{\beta}, \hat{\mu}\right)$, and for $\mu \in (\hat{\mu}, \bar{\mu})$ (see Corollary 2 in the appendix). Intuitively, the real value of money falls as μ increases, and the marginal investor type, ε^*, decreases, reflecting the fact that under the higher inflation rate, the investor that was marginal under the lower inflation rate is no longer indifferent between carrying cash and equity out of the OTC market (he prefers equity).

According to Proposition 3, $\phi^s y_t < (\phi^s + \varepsilon^*) y_t = p_t \phi_t^m$ in the monetary equilibrium, so Lemma 1 implies that dealers hold no equity shares at the end of the OTC round: all equity is held by investors, in particular, by those investors who carried equity into the period but were unable to contact a dealer, and by those investors who purchased equity shares from dealers. After the round of OTC trade, all the money supply is held by the investors who carried cash into the period but were unable to contact a dealer, by the investors who sold equity shares to dealers, and by those dealers who had bargaining power in the OTC negotiations or carried equity into the OTC market. A feature of the monetary equilibrium is that dealers never hold money overnight: at the beginning of every period t, the money supply is all in the hands of investors, i.e., $A^m_{Dt} = 0$ and $A^m_{It} = A^m_t$.[11] The reason is that access to the interdealer market allows dealers to intermediate assets without having to carry cash. Whether it is investors or dealers who hold the equity overnight, depends on the inflation rate: if it is low, i.e., if $\mu \in (\bar{\beta}, \hat{\mu})$, then only dealers hold equity shares overnight, that is, $\tilde{A}^s_{Dt+1} = A^s$ and $\tilde{A}^s_{It+1} = 0$ for all t. Conversely, if the inflation rate is high, i.e., if $\mu \in (\hat{\mu}, \bar{\mu})$, then at the end of every period t, all equity shares are in the hands of investors, i.e., $\tilde{A}^s_{Dt+1} = 0$ and $\tilde{A}^s_{It+1} = A^s$. To understand this result, it is useful to inspect the Euler equations for equity shares. In a stationary equilibrium, (20) reduces to

$$\begin{aligned} 1 &\geq \beta\pi\mathbb{E}_t \frac{\max\left(p_{t+1}\phi^m_{t+1}, \phi^s_{t+1}\right)}{\phi^s_t} \\ 1 &\geq \beta\pi R^s_D\left(\varepsilon^*\right), \text{ “ = ” if } \tilde{A}^s_{Dt+1} > 0, \end{aligned} \tag{33}$$

where

$$R^s_D\left(\varepsilon^*\right) \equiv \mathbb{E}_t \frac{\max\left(p_{t+1}\phi^m_{t+1}, \phi^s_{t+1}\right)}{\phi^s_t} = \frac{\varepsilon^* + \phi^s}{\phi^s}\bar{\gamma}.$$

Dealers do not wish to hold equity overnight if (33) holds with strict inequality. The equilibrium return to a dealer from holding equity overnight, $R^s_D\left(\varepsilon^*\right)$, consists of the expected capital gain from purchasing equity in the second subperiod and reselling it in the OTC market of the following period. Similarly, in a stationary equilibrium (22) reduces to

$$1 \geq \beta\pi R^s_I\left(\varepsilon^*\right), \text{ “ = ” if } \tilde{A}^s_{It+1} > 0, \tag{34}$$

where

$$\begin{aligned} R^s_I\left(\varepsilon^*\right) &\equiv \mathbb{E}_t \frac{\left[\bar{\varepsilon} + \phi^s + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon)dG\left(\varepsilon\right)\right]y_{t+1}}{\phi^s y_t} \\ &= G\left(\varepsilon^*\right)\left\{\delta\theta\frac{\bar{\phi}^s}{\phi^s}\bar{\gamma} + \left[(1-\delta) + \delta\left(1-\theta\right)\right]\frac{\bar{\varepsilon}^{*l} + \phi^s}{\phi^s}\bar{\gamma}\right\} + \left[1 - G(\varepsilon^*)\right]\frac{\bar{\varepsilon}^{*h} + \phi^s}{\phi^s}\bar{\gamma}, \end{aligned}$$

$\bar{\varepsilon}^{*l} \equiv \int_{\varepsilon_L}^{\varepsilon^*}\varepsilon\frac{dG(\varepsilon)}{G(\varepsilon^*)}$, and $\bar{\varepsilon}^{*h} \equiv \int_{\varepsilon^*}^{\varepsilon_H}\varepsilon\frac{dG(\varepsilon)}{1-G(\varepsilon^*)}$. Investors do not wish to hold equity overnight if (34) holds with strict inequality. The equilibrium expected return to an investor from holding

[11] In a stationary equilibrium, (19) becomes $\mu\bar{\phi}^s > \bar{\beta}\max\left(\bar{\phi}^s, \phi^s\right) = \bar{\beta}\bar{\phi}^s$.

equity overnight, $R_I^s(\varepsilon^*)$, can be thought of as a weighted average of four gross returns. The first, $\frac{\bar{\phi}^s}{\phi^s}\bar{\gamma}$, is the expected capital gain of an investor who sells equity in the OTC market at the interdealer market price ($\bar{\phi}^s$), which is what occurs with probability $G(\varepsilon^*)\,\delta\theta$, i.e., when the investor draws a preference type lower than ε^* (so he wishes to sell in the OTC market), contacts a dealer, and has the bargaining power. The second, $\frac{\bar{\varepsilon}^{*l}+\phi^s}{\phi^s}\bar{\gamma}$, is the expected equity return to an investor who wishes to sell in the OTC market but fails to contact a dealer, which occurs with probability $G(\varepsilon^*)\,(1-\delta)$. In this case the expected equity payoff consists of the expected value of the period dividend conditional on wanting to sell, $\mathbb{E}_t\bar{\varepsilon}^{*l}y_{t+1}$, and the expected resale value of the equity in the following Walrasian round of trade, $\mathbb{E}_t\phi^s y_{t+1}$. The third, also $\frac{\bar{\varepsilon}^{*l}+\phi^s}{\phi^s}\bar{\gamma}$, is the expected capital gain of an investor who sells equity in the OTC market at the dealer's expected bid price, $\mathbb{E}_t(\bar{\varepsilon}^{*l}+\phi^s)y_{t+1}$, that (in expected value) reaps all the gains from trade from an investor who wishes to sell, an event that occurs with probability $G(\varepsilon^*)\,\delta\theta$, i.e., when the investor draws a preference type lower than ε^* (so he wishes to sell in the OTC market), contacts a dealer, and the dealer has the bargaining power. The fourth, $\frac{\bar{\varepsilon}^{*h}+\phi^s}{\phi^s}\bar{\gamma}$, is the expected equity return of an investor who does not wish to sell in the OTC market and therefore keeps the equity share for a full period, which occurs with probability $1-G(\varepsilon^*)$. In this case the expected equity payoff consists of the expected value of the period dividend conditional on not wanting to sell, $\mathbb{E}_t\bar{\varepsilon}^{*h}y_{t+1}$, and the resale value of the equity in the following Walrasian round of trade, $\mathbb{E}_t\phi^s y_{t+1}$.

From (33) and (34), dealers hold all equity shares overnight (i.e., $\tilde{A}^s_{Dt+1}=A^s$ and $\tilde{A}^s_{It+1}=0$) if and only if $R_I^s(\varepsilon^*)<R_D^s(\varepsilon^*)$, i.e., if and only if $\bar{\varepsilon}+\delta\theta G(\varepsilon^*)(\varepsilon^*-\bar{\varepsilon}^{*l})<\varepsilon^*$. This condition is equivalent to

$$\bar{\varepsilon}+\delta\theta\int_{\varepsilon_L}^{\varepsilon^*}G(\varepsilon)\,d\varepsilon<\varepsilon^*,$$

which is in turn equivalent to $\hat{\varepsilon}<\varepsilon^*$, where $\hat{\varepsilon}$ is defined by (25). Intuitively then, all equity is held by dealers overnight if the marginal preference type that partitions investors between buyers and sellers is large enough (larger than $\hat{\varepsilon}$), else all equity is held by investors overnight, and strictly speaking, dealers only provide brokerage services in the OTC market.[12]

[12] To get more intuition, write $R_I^s(\varepsilon^*)=[\bar{\varepsilon}+\delta\theta G(\varepsilon^*)(\varepsilon^*-\bar{\varepsilon}^{*l})+\phi^s]\bar{\gamma}/\phi^s$ and notice that $\partial R_I^s(\varepsilon^*)/\partial\varepsilon^*=\delta\theta G(\varepsilon^*)\bar{\gamma}/\phi^s<\bar{\gamma}/\phi^s=\partial R_D^s(\varepsilon^*)/\partial\varepsilon^*$ and

$$\frac{\varepsilon_L+\phi^s}{\phi^s}\bar{\gamma}=R_D^s(\varepsilon_L)<R_I^s(\varepsilon_L)=\frac{\bar{\varepsilon}+\phi^s}{\phi^s}\bar{\gamma}<\frac{[\delta\theta\varepsilon_H+(1-\delta\theta)\bar{\varepsilon}]+\phi^s}{\phi^s}\bar{\gamma}=R_I^s(\varepsilon_H)<R_D^s(\varepsilon_H)=\frac{\varepsilon_H+\phi^s}{\phi^s}\bar{\gamma}.$$

This reasoning is in terms of ε^* while Proposition 3 is stated in terms of μ, but there is a monotonic relationship between μ and ε^*. In the proof of this proposition it is shown that $\hat{\varepsilon}<\varepsilon^*$ if and only if $\mu<\hat{\mu}$.

Given the marginal preference type, ε^*, part (iv) of Proposition 3 gives all asset prices in closed form. The real price of equity (in terms of the homogeneous consumption good) in the Walrasian round of trade, ϕ_t^s, is given by (32). The dollar price of equity in the OTC market, p_t, is given by (30). The real price of money (in terms of the homogeneous consumption good) in the Walrasian round of trade, ϕ_t^m, is given by (29). The real price of equity (in terms of the homogeneous consumption good) in the OTC market, $p_t\phi_t^m = \bar{\phi}^s y_t$ is given by (28).

Finally, part $(v)(a)$ states that as the rate of money creation rises toward $\bar{\mu}$, ε^* approaches the lower bound of the type distribution, ε_L, so no investor wishes to sell equity in the OTC market, and as a result the allocations and prices of the monetary equilibrium approach those of the nonmonetary equilibrium. Part $(v)(b)$ states that as the rate of money creation falls toward $\bar{\beta}$, ε^* increases toward the upper bound of the type distribution, ε_H, so only investors with the highest preference type purchase equity in the OTC market (all other investors wish to sell it).

4.2 Non-intermediated OTC market

In this section we consider the case with $\delta = 0$, i.e., a market in which there are no specialized dealers and all OTC trade is conducted bilaterally among investors. Let

$$\tilde{\mu} \equiv \bar{\beta}\left[1 + \alpha\eta \int_{\varepsilon_L}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}} dG(\varepsilon_i)\, dG(\varepsilon_j)\right], \tag{35}$$

and define the function $\varphi : [\varepsilon_L, \varepsilon_H] \to \mathbb{R}$ by

$$\varphi(\varepsilon) \equiv \int_{\varepsilon_L}^{\varepsilon}\int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i)\, dG(\varepsilon_i)\, dG(\varepsilon_j)\,.$$

Proposition 4 *Assume $\delta = 0$. (i) Dealers are inactive in any equilibrium, and a nonmonetary equilibrium exists for any parametrization. (ii) There is no stationary monetary equilibrium if $\mu \geq \tilde{\mu}$. (iii) In the nonmonetary equilibrium there is no trade in the OTC market, and the equity price in the Walrasian market is*

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y.$$

(iv) If $\mu \in (\bar{\beta}, \tilde{\mu})$, then there is one stationary monetary equilibrium and asset prices are

$$\phi_t^s = \phi^s y_t, \text{ with } \phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon^c)\right] \tag{36}$$

$$\phi_t^m = Z\frac{y_t}{A_t^m}$$

where

$$Z = (\varepsilon^c + \phi^s) A^s, \tag{37}$$

and for any $\mu \in \left(\bar{\beta}, \tilde{\mu}\right)$, $\varepsilon^c \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(1-\bar{\beta}\pi)(\varepsilon_i - \varepsilon_j)}{(1-\bar{\beta}\pi)\varepsilon_j + \bar{\beta}\pi\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon^c)\right]} dG(\varepsilon_i)\, dG(\varepsilon_j) - \frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\eta} = 0. \tag{38}$$

(v) (a) As $\mu \to \tilde{\mu}$, $\varepsilon^c \to \varepsilon_L$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y_t$. *(b) As* $\mu \to \bar{\beta}$, $\varepsilon^c \to \varepsilon_H$ *and* $\phi_t^s \to \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon_H)\right] y_t$.

The interdealer market is inactive when $\delta = 0$, and so are dealers. A stationary monetary equilibrium does not exist if the inflation rate is too large, i.e., if $\mu \geq \tilde{\mu}$, and in this case there is no equity trade and the equity price is equal to the expected discounted present value of the dividend. If the inflation rate is low enough, i.e., $\mu \in (\bar{\beta}, \tilde{\mu})$, then a unique stationary equilibrium exists. In this case an investor i starts every period t with a portfolio of money and equity, and he is randomly matched with another investor j during the OTC round of trade. If i's preference shock is larger than j's, i.e., if $\varepsilon_j < \varepsilon_i$, then i will want to purchase all of j's equity holdings. Whether he is able to do so depends the quantity of assets that j holds, which in equilibrium equals A^s, and the dollar price that i has to pay for the equity. In turn, the dollar price will depend on whether investor i or investor j has the bargaining power. If i has the power (this happens with probability η), then the dollar price he pays j for each unit of equity is $p_t^o(\varepsilon_j) = \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m}$, and since i holds A_t^m dollars in equilibrium, he can afford to buy all of j's equity only if $\frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} A^s \leq A_t^m$, or equivalently, if $(\varepsilon_j + \phi^s) A^s \leq Z$ which using (37) can be rewritten as $\varepsilon_j \leq \varepsilon^c$. Thus as μ falls, real balances and ε^c increase, and the investor who wishes to buy equity and has the bargaining power, is cash constrained in a smaller fraction of the bilateral meetings. The value of money depends on the gains from trade of the relatively high valuation investors who buy equity. The equity price (36) on the other hand, reflects the gains from trade of the relatively low valuation investors who sell equity in bilateral transactions ($\alpha(1-\eta)\varphi(\varepsilon^c)$ captures the expected gain from selling equity to another investor in the OTC round).

5 Asset prices

In this section we study the properties of the equilibrium asset prices in Proposition 3 and Proposition 4. In particular, we focus on how they depend on monetary policy and the degree of OTC frictions as captured by the parameters that regulate trading frequency and the relative bargaining strengths of the various traders.

5.1 Inflation

In a pure-dealer OTC market, the real price of equity in a monetary equilibrium is in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. As μ increases, equilibrium real money balances fall and the marginal investor type, ε^*, decreases reflecting the fact that under the higher inflation rate, the investor type that was marginal under the lower inflation rate is no longer indifferent between carrying cash and equity out of the OTC market (he prefers equity). Since the marginal investor who prices the equity in the OTC market has a lower valuation, the value of the resale option is smaller, which in turn makes the real equity price (both ϕ^s and $\bar{\phi}^s$) smaller. As expected, the real value of money, ϕ_t^m, also declines with the rate of inflation. These arguments are formalized in Proposition 5. The top row of Figure 1 illustrates the time paths of the ex-dividend equity price, ϕ_t^s, real balances $\phi_t^m A_t^m$, and the price level, ϕ_t^m for different values of μ.

Proposition 5 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\mu < 0$, (ii) $\partial\bar{\phi}^s/\partial\mu < 0$, (iii) $\partial Z/\partial\mu < 0$ and $\partial\phi_t^m/\partial\mu < 0$.*

In a non-intermediated OTC market, the real price of equity in a monetary equilibrium is also in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. A higher inflation rate causes real money balances to decline. This reduction in real balances enlarges the set of joint realizations of preference types in bilateral meetings in which the cash constraint binds for high-valuation buyers. In turn, this reduces the value of the marginal preference type, ε^c, of the buyer who is just able to purchase all of the equity from a seller in a bilateral meeting in which the seller has the bargaining power. The result is that ex ante, the period before the OTC round, investors anticipate that the expected gains from selling equity in the OTC market are smaller, and this manifests itself as a smaller equity price in the centralized round of trade. The following proposition formalizes this intuition.

Proposition 6 *Consider the formulation with $\delta = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\mu < 0$, (ii) $\partial Z/\partial\mu < 0$ and $\partial\phi_t^m/\partial\mu < 0$.*

5.2 OTC frictions: trading delays and market power

In a pure-dealer OTC market, $\delta\theta$ is an investor's effective bargaining power in negotiations with dealers. A larger $\delta\theta$ implies a larger gain from trade for low-valuation investors when they off-load the asset onto dealers in the OTC market. This in turn makes investors more willing to hold equity shares in the centralized market of the previous period, since they anticipate larger gains from selling the equity in case they were to draw a relatively low preference type in the following OTC round. As a result, real equity prices, ϕ^s and $\bar{\phi}^s$, are increasing in δ and θ. If δ increases, money becomes more valuable (both Z and ϕ_t^m increase), provided we focus on a regime in which only investors carry equity overnight.[13] Proposition 7 formalizes these ideas. The bottom row of Figure 1 illustrates the time paths of the ex-dividend equity price, ϕ_t^s, real balances $\phi_t^m A_t^m$, and the price level, ϕ_t^m for two different values of δ.

Proposition 7 *Consider the formulation with* $\alpha = 0$. *In the stationary monetary equilibrium: (i)* $\partial\phi^s/\partial(\delta\theta) > 0$, *(ii)* $\partial\bar{\phi}^s/\partial(\delta\theta) > 0$, *(iii)* $\partial Z/\partial\delta > 0$ *and* $\partial\phi_t^m/\partial\delta > 0$, *for* $\mu \in (\hat{\mu}, \bar{\mu})$.

The following proposition establishes that in a non-intermediated OTC market, the value of holding equity increases with the bilateral meeting probability, as this increases the probability that the investor may find an opportunity to sell the asset to another investor with higher valuation. Similarly, the value of money increases with α as this increases the probability the investor may be able to use money to buy equity if he were to meet a counterparty with lower valuation.

Proposition 8 *Consider the formulation with* $\delta = 0$. *In the stationary monetary equilibrium: (i)* $\partial\phi^s/\partial\alpha > 0$, *(ii)* $\partial Z/\partial\alpha > 0$ *and* $\partial\phi_t^m/\partial\alpha > 0$.

6 Financial liquidity

In this section we use the theory to study the determinants of standard measures of market liquidity: liquidity provision by dealers, trade volume, and bid-ask spreads. Broker-dealers in OTC markets provide liquidity (*immediacy*) to investors by finding them counterparties for trade, and/or by trading with them out of their own account, effectively becoming their counterparty. Trade volume is a manifestation of the ability of the OTC market to reallocate

[13] Real balances can actually fall with δ for $\mu \in (\bar{\beta}, \hat{\mu})$.

assets across investors. Bid-ask spreads constitute the main out-of-pocket transaction cost that investors bear in OTC markets. Section 6.1 focuses on dealers' decisions to hold asset inventories with the purpose of becoming trade counterparties for investors. Section 6.2 focuses on the determinants of trade volume. Bid-ask spreads are analyzed in Section 6.3.

6.1 Liquidity provision by dealers

To simplify the exposition, here we focus on the formulation with $\alpha = 0$. The following result characterizes the effect of inflation on dealers' provision of liquidity by accumulating assets.

Proposition 9 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) dealers' provision of liquidity by accumulating assets, i.e., A_D^s, is nonincreasing in the inflation rate. (ii) For any μ close to $\bar{\beta}$, dealers' provision of liquidity by accumulating assets is nonmonotonic in $\delta\theta$, i.e., $A_D^s = 0$ for $\delta\theta$ close to 0 and close to 1, but $A_D^s > 0$ for intermediate values of $\delta\theta$.*

To understand part (i) of Proposition 9, recall the discussion that followed Proposition 3. The expected return from holding equity is larger for investors than for dealers with high inflation ($\mu > \hat{\mu}$) because in that case the expected resale value of equity in the OTC market is relatively low, and dealers only buy equity to resell in the OTC market, while investors also buy it with the expectation of getting utility from the dividend flow. For low inflation ($\mu < \hat{\mu}$), dealers value equity more than investors because the OTC resale value is high and they have a higher probability of making capital gains from reselling than investors, and this trading advantage more than compensates for the fact that investors enjoy the additional utility from the dividend flow. Part (ii) of Proposition 9 states that given a low enough rate of inflation, dealers' incentives to hold equity inventories overnight depend nonmonotonically on the degree of OTC frictions as measured by $\delta\theta$. In particular, dealers will not hold inventories if $\delta\theta$ is either very small or very large. If $\delta\theta$ is close to zero, few investors contact the interdealer market, and this makes the equity price in the OTC market very low, which in turn implies too small a capital gain to induce dealers to hold equity overnight. Conversely, if $\delta\theta$ is close to one, a dealer has no trading advantage over an investor in the OTC market and since the investor gets utility from the dividend while the dealer does not, the investor has a higher willingness to pay for the asset in the centralized market than the dealer, and therefore it is investors and not dealers who carry the asset overnight into the OTC market.

6.2 Volume

First consider the case with $\alpha = 0$. According to Lemma 2, any investor with $\varepsilon < \varepsilon_t^*$ who has a trading opportunity in the OTC market, sells all his equity. Hence in a stationary equilibrium, the quantity of assets sold by investors to dealers in the OTC market is $Q^- = \delta G(\varepsilon^*) A_I^s$. From Lemma 2 we also know that an investor with $\varepsilon > \varepsilon_t^*$ who contacts a dealer will buy equity, and that the quantity he buys depends on whether the investor or the dealer has the bargaining power. If the investor has the bargaining power then he purchases A_t^m/p_t equity shares. Thus the volume of assets traded by such investors is $Q^{+*} = \delta\theta[1 - G(\varepsilon^*)] A_t^m/p_t$. If instead the dealer has the bargaining power, then the investor purchases $A_t^m/p_t^o(\varepsilon)$ equity shares. Therefore the volume of assets traded by such investors is $Q^+ = \delta(1-\theta)\int_{\varepsilon^*}^{\varepsilon_H}[A_t^m/p_t^o(\varepsilon)]\,dG(\varepsilon)$. The total quantity of equity shares traded in the OTC market is $\mathcal{V} = Q^- + Q^{+*} + Q^+$, or equivalently[14]

$$\mathcal{V} = \pi\tilde{A}_D^s + 2\delta G(\varepsilon^*)(A^s - \pi\tilde{A}_D^s). \tag{39}$$

Trade volume $\mathcal{V}$ depends on μ and θ only indirectly, through ε^*. A decrease in μ or an increase in θ, increases the expected return to holding money, which makes more investors willing to sell equity for money in the OTC market, i.e., ε^* increases and so does trade volume. The indirect (through ε^*) positive effect on $\mathcal{V}$ of an increase in the investors' trade probability δ is similar to an increase in θ, but in addition, δ directly increases trade volume since with a higher δ more investors are able to trade in the OTC market. These results are summarized in the following proposition.

Proposition 10 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\mathcal{V}/\partial\mu < 0$, (ii) $\partial\mathcal{V}/\partial\theta > 0$ and $\partial\mathcal{V}/\partial\delta > 0$.*

Next consider the formulation with $\delta = 0$. According to Lemma 3, the quantity traded in a meeting between two investors depends on whether the buyer or the seller of equity has the bargaining power. Suppose that investor i has preference type ε_i and investor j has preference type $\varepsilon_j < \varepsilon_i$. If investor i (in this case the buyer) makes the offer, then he purchases $\min\{A_t^m/p_t^o(\varepsilon_j), A^s\} = \mathbb{I}_{\{\varepsilon^c<\varepsilon_j\}}\frac{Z}{\varepsilon_j+\phi^s} + \mathbb{I}_{\{\varepsilon_j\leq\varepsilon^c\}}A^s$ equity shares. Conversely, if investor j has the bargaining power, then investor i purchases $\min\{A_t^m/p_t^o(\varepsilon_i), A^s\} =$

[14]To obtain (39) we used the clearing condition for the interdealer market, $A_D^s + Q^- = Q^+ + Q^{+*}$ which implies $\mathcal{V} = Q^- + Q^{+*} + Q^+ = A_D^s + 2Q^-$. Also, note that $\mathcal{V}$ is trade volume in the OTC market, but since every equity share traded in the first subperiod gets retraded in the second subperiod, total trade volume in the whole time period equals $2\mathcal{V}$.

$\mathbb{I}_{\{\varepsilon^c<\varepsilon_i\}}\frac{Z}{\varepsilon_i+\phi^s}+\mathbb{I}_{\{\varepsilon_i\leq\varepsilon^c\}}A^s$ equity shares. Hence the total quantity of equity shares traded in the OTC market is

$$\tilde{\mathcal{V}}=\alpha\left\{\eta\left[\int_{\varepsilon_L}^{\varepsilon^c}\left[1-G\left(\varepsilon_j\right)\right]dG\left(\varepsilon_j\right)+\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon^c+\phi^s}{\varepsilon_j+\phi^s}dG\left(\varepsilon_i\right)dG\left(\varepsilon_j\right)\right]\right.$$
$$\left.+\left(1-\eta\right)\left[\int_{\varepsilon_L}^{\varepsilon^c}G\left(\varepsilon_i\right)dG\left(\varepsilon_i\right)+\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_L}^{\varepsilon_i}\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}dG\left(\varepsilon_j\right)dG\left(\varepsilon_i\right)\right]\right\}A^s.$$

In trades where the investor with no bargaining power has preference type $\varepsilon<\varepsilon^c$, all the equity holdings of the seller, A^s, are traded. In meetings where the investor with no bargaining power has preference type $\varepsilon>\varepsilon^c$, the cash constraint of the buyer binds, and only $\frac{Z}{\varepsilon_j+\phi^s}=\frac{(\varepsilon^c+\phi^s)A^s}{\varepsilon_j+\phi^s}$ equity shares are traded. Notice that inflation only affects $\tilde{\mathcal{V}}$ indirectly, through its effect on ε^c (or equivalently, real balances, Z). Higher inflation reduces the value of real balances and this implies that the cash constraint will bind in more trades, causing trade volume to decline along the intensive margin (i.e., by reducing the quantity of equity traded in trades in which the agent with no bargaining power has relatively high valuation for the dividend). An increase in the contact probability α increases $\tilde{\mathcal{V}}$ along the extensive margin (more meetings among investors naturally result in larger trade volume), but an increase in α also increases real balances and therefore induces an increase in trade volume along the intensive margin. This intuition is formalized in the following proposition.

Proposition 11 *Consider the formulation with $\delta=0$. In the stationary monetary equilibrium: (i) $\partial\tilde{\mathcal{V}}/\partial\mu<0$, and (ii) $\partial\tilde{\mathcal{V}}/\partial\alpha>0$.*

6.3 Spreads

Focus on the formulation with $\alpha=0$. Corollary 1 shows that when dealers with bargaining power execute trades on behalf of their investors, they charge an ask price $p_t^o\left(\varepsilon\right)>p_t$ to investors with $\varepsilon>\varepsilon^*$ who wish to buy equity, and pay a bid price $p_t^o\left(\varepsilon\right)<p_t$ to investors with $\varepsilon<\varepsilon^*$ who wish to sell equity. Thus in any transaction with an investor with preference type ε, the dealer earns a nominal spread $\mathcal{S}_t^m\left(\varepsilon\right)=\left|p_t^o\left(\varepsilon\right)-p_t\right|$. Define the real spread $\mathcal{S}\left(\varepsilon\right)=\mathcal{S}_t^m\left(\varepsilon\right)/p_t$, i.e.,

$$\mathcal{S}\left(\varepsilon\right)=\frac{\left|\varepsilon-\varepsilon^*\right|}{\varepsilon^*+\phi^s}.$$

The average real spread is $\bar{\mathcal{S}}=\int\mathcal{S}\left(\varepsilon\right)dG\left(\varepsilon\right)$, i.e.,

$$\bar{\mathcal{S}}=\frac{1}{\varepsilon^*+\phi^s}\left[\int_{\varepsilon_L}^{\varepsilon^*}\left(\varepsilon^*-\varepsilon\right)dG\left(\varepsilon\right)+\int_{\varepsilon^*}^{\varepsilon_H}\left(\varepsilon-\varepsilon^*\right)dG\left(\varepsilon\right)\right].$$

The change in the average spread in response to changes in μ, δ or θ, are ambiguous in general. The reason is that $S(\varepsilon)$ is decreasing in ε^* for buyers with $\varepsilon > \varepsilon^*$, but may be increasing in ε^* for investors who wish to sell (i.e., those with $\varepsilon < \varepsilon^*$).

6.4 Speculation

According to Proposition 3 and Proposition 4, in a monetary equilibrium the equity price, ϕ^s, is larger than the expected present discounted value that any agent assigns to the dividend stream, i.e., $\hat{\phi}_t^s \equiv \left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\bar{\varepsilon}y_t$. It is commonplace to define the *fundamental value* of the asset to be the expected present discounted value of the dividend stream, and to call any transaction value in excess of this fundamental value, a *bubble*.[15] One could argue, of course, that the relevant notion of "fundamental value" should be calculated through market aggregation of diverse investor valuations, and taking into account the monetary policy stance as well as all the details of the market structure in which the asset is traded (such as the frequency of trading opportunities and the degree of market power of financial intermediaries) which ultimately also factor into the asset price in equilibrium. In any case, to avoid semantic controversies, we follow Harrison and Kreps (1978) and call the value of the asset in excess of the expected present discounted value of the dividend, i.e., $\phi_t^s - \hat{\phi}_t^s$, the "speculative premium" which investors are willing to pay in anticipation of the capital gains they will reap when reselling the asset to investors with higher valuations in the future. So like Harrison and Kreps (1978), we say that investors exhibit *speculative behavior* if the right to resell a stock makes them willing to pay more for it than they would pay if obliged to hold it forever. Investors exhibit speculative behavior in the sense that they buy with the expectation to resell, and naturally the asset price incorporates the value of this option to resell: investors are willing to pay more for the asset than they would pay if obliged to hold it forever.

Consider the case pure-dealer case (i.e., $\alpha = 0$). According to Proposition 3, in a monetary equilibrium the speculative premium is $\mathcal{P}_t = \mathcal{P}y_t$, where

$$\mathcal{P} = \begin{cases} \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left(\varepsilon^* - \bar{\varepsilon}\right) & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

The speculative premium is positive in any monetary equilibrium, i.e., $\mathcal{P}_t \geq 0$, with "=" only if $\mu = \bar{\mu}$. Since $\partial\varepsilon^*/\partial\mu < 0$ (Corollary 2), it is immediate that the speculative premium is decreas-

[15]See, e.g., Barlevy (2007), Scheinkman and Xiong (2003), and Xiong (2013) who discusses Harrison and Kreps' paper in the context of what he calls the "resale option theory of bubbles."

ing in the rate of inflation. Intuitively, anticipated inflation reduces the real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium is decreasing in μ. Since $\partial\varepsilon^*/\partial(\delta\theta) > 0$ (see the proof of Proposition 7), the speculative premium is increasing in δ and θ. Intuitively, the speculative premium is the value of the option to resell the equity to a higher valuation investor in the future, and the value of this resale option to the investor increases with the probability δ that the investor gets a trading opportunity in an OTC trading round and with the probability θ that he can capture the gains from trade in those trades. Interestingly, notice that the signs of these effects are exactly the opposite than they are for trade volume. So in low-inflation regimes, the model predicts large trade volume and a large speculative premium. Figure 2 illustrates the time path of the speculative premium $\mathcal{P}_t$ for two values of μ. The following proposition summarizes these results.

Proposition 12 *Consider the formulation with* $\alpha = 0$*. In the stationary monetary equilibrium: (i)* $\partial\mathcal{P}/\partial\mu < 0$*, and (ii)* $\partial\mathcal{P}/\partial(\delta\theta) > 0$*.*

Consider a non-intermediated OTC market (i.e., $\delta = 0$). According to Proposition 4, in a monetary equilibrium the speculative premium is $\tilde{\mathcal{P}}_t = \tilde{\mathcal{P}}y_t$, where

$$\tilde{\mathcal{P}} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\alpha(1-\eta)\varphi(\varepsilon^c).$$

Again, $\tilde{\mathcal{P}}_t \geq 0$, with "=" only if $\mu = \bar{\mu}$. Higher inflation reduces real balances and therefore ε^c, which reduces the expected resale value of equity in the OTC market, so $\tilde{\mathcal{P}}$ decreases with inflation. An increase in the trade probability α has a positive direct effect on $\tilde{\mathcal{P}}$ (increase in the meeting probability) and also an indirect positive effect (α increases ε^c which in turn increases $\tilde{\mathcal{P}}$). These effects are summarized below.

Proposition 13 *Consider the formulation with* $\delta = 0$*. In the stationary monetary equilibrium: (i)* $\partial\tilde{\mathcal{P}}/\partial\mu < 0$*, and (ii)* $\partial\mathcal{P}/\partial\alpha > 0$*.*

7 Endogenous trading delays

In this section we endogenize the supply of intermediation services and the length of the trading delays by allowing for free entry of dealers. This formalizes the notion that a dealer's profit depends on the competition for order flow that he faces from other dealers.

In this section we use δ to denote a continuously differentiable function of the measure of dealers in the market, v, i.e., $\delta : \mathbb{R}_{+} \to [0,1]$, and let $\kappa(v) \equiv \delta(v)/v$. We assume $\delta'(v) > 0$, $\kappa'(v) < 0$, and $\delta''(v) < 0$. We also specify $\delta(0) = \lim_{v\to\infty} \kappa(v) = 0$, and $\lim_{v\to\infty} \delta(v) = \kappa(0) = 1$. These assumptions capture the notion that if the measure of dealers, v, is larger, then each investor contacts dealers faster, while the order flow for each individual dealer decreases.[16] There is a large measure of dealers who can choose to participate in the market. Dealers who wish to provide intermediation services in the OTC market of period $t+1$ must incur a real resource cost k_t in the second subperiod of period t, i.e., right before they can participate in the OTC round of trade. This entry cost is in terms of the homogeneous good and represents the ongoing expenses of running the dealership business. To keep the environment stationary, we assume $k_t = ky_t$ for some $k \in \mathbb{R}_{++}$.

7.1 Efficiency

With the notation introduced in Section 3, the planner's problem for the economy with free entry (and $\alpha = 0$) consists of choosing a nonnegative allocation

$$\left\{v_t, \tilde{a}_{tD}, a'_{tD}, c_{tD}, h_{tD}, \tilde{a}_{tI}, a'_{tI}, [c_{tI}(\varepsilon_i), h_{tI}(\varepsilon_i)]_{\varepsilon\in[\varepsilon_L,\varepsilon_H]}\right\}_{t=0}^{\infty},$$

to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\delta(v_t) \int_{[\varepsilon_L,\varepsilon_H]} \varepsilon y_t a'_{tI}(d\varepsilon) + [1-\delta(v_t)]\,\bar{\varepsilon} y_t a_{tI} + \int_{\varepsilon_L}^{\varepsilon_H} [c_{tI}(\varepsilon) - h_{tI}(\varepsilon)]\,dG(\varepsilon) + (c_{tD} - h_{tD})v_t\Bigg]$$

subject to

$$v_t \tilde{a}_{tD} + \tilde{a}_{tI} \le A^s \tag{40}$$

$$v_t a'_{tD} + \delta(v_t) \int_{[\varepsilon_L,\varepsilon_H]} a'_{tI}(d\varepsilon) \le v_t a_{tD} + \delta(v_t)\, a_{tI} \tag{41}$$

$$\int_{\varepsilon_L}^{\varepsilon_H} c_{tI}(\varepsilon)\,dG(\varepsilon) + c_{tD} v_t + k_t v_{t+1} \le \int_{\varepsilon_L}^{\varepsilon_H} h_{tI}(\varepsilon)\,dG(\varepsilon) + v_t h_{tD}, \tag{42}$$

(6) and (7). The following proposition characterizes the solution to the planner's problem.

[16] The matching function $\delta(v) = 1 - e^{-v}$ is an example that satisfies the maintained assumptions.

Proposition 14 *The efficient allocation for the model with free entry of dealers has*

$$-ky_t+\beta\mathbb{E}_t\delta'\left(v_{t+1}\right)\left(\varepsilon_H-\bar{\varepsilon}\right)y_{t+1}\left(1-\pi\right)A^s\leq 0,\text{ “}=\text{” if } v_{t+1}>0, \tag{43}$$

$\tilde{a}_{tD}=(A^s-\tilde{a}_{tI})/v_t=A^s/v_t$, *and* $a'_{ti}(E)=\mathbb{I}_{\{\varepsilon_H\in E\}}\left[\pi/\delta\left(v_t\right)+(1-\pi)\right]A^s$, *where* $\mathbb{I}_{\{\varepsilon_H\in E\}}$ *is an indicator function that takes the value* 1 *if* $\varepsilon_H\in E$, *and* 0 *otherwise, for any* $E\in\mathcal{F}\left([\varepsilon_L,\varepsilon_H]\right)$.

7.2 Equilibrium

An equilibrium with free entry is characterized by the same equations that characterize an equilibrium in the baseline model of Section 2 (replacing δ with $\delta\left(v_t\right)$ and κ with $\kappa\left(v_t\right)$), plus the following condition, which must hold in an equilibrium in which dealers are free to participate in the OTC of any period t

$$W_t^D\left(\mathbf{0}\right)-k_t\leq 0,\text{ with “}=\text{” if } v_{t+1}>0. \tag{44}$$

For each t, the free-entry condition (44) can be used to determine the additional unknown v_{t+1}. Lemma 8 in the appendix shows that (44) can be written as

$$\Phi_{t+1}-k_t\leq 0,\text{ with “}=\text{” if } v_{t+1}>0 \tag{45}$$

for all t, where

$$\begin{aligned}\Phi_{t+1}\equiv\beta\mathbb{E}_t\kappa\left(v_{t+1}\right)\left(1-\theta\right)\bar{\phi}_{t+1}&\left[\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\frac{(\varepsilon_{t+1}^*-\varepsilon)y_{t+1}}{\varepsilon_{t+1}^*y_{t+1}+\phi_{t+1}^s}p_{t+1}A_{It+1}^s dG\left(\varepsilon\right)\right.\\&\left.+\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\frac{(\varepsilon-\varepsilon_{t+1}^*)y_{t+1}}{\varepsilon y_{t+1}+\phi_{t+1}^s}A_{It+1}^m dG\left(\varepsilon\right)\right]\end{aligned} \tag{46}$$

is a dealer's discounted expected income from intermediation in the OTC market of period $t+1$. In order to interpret Φ_{t+1}, it is useful to define $Q_{t+1}\left(\varepsilon\right)\equiv\frac{1}{p_{t+1}^o(\varepsilon)}A_{It+1}^m$,

$$\begin{aligned}\mathcal{S}_{t+1}^b&\equiv\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}[p_{t+1}-p_{t+1}^o\left(\varepsilon\right)]A_{t+1}^I\frac{dG\left(\varepsilon\right)}{G\left(\varepsilon_{t+1}^*\right)}\\\mathcal{S}_{t+1}^a&\equiv\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}[p_{t+1}^o\left(\varepsilon\right)-p_{t+1}]Q_{t+1}\left(\varepsilon\right)\frac{dG\left(\varepsilon\right)}{1-G\left(\varepsilon_{t+1}^*\right)},\end{aligned}$$

where $p_{t+1}^o\left(\varepsilon\right)$ (defined in (16)) is the nominal equity price that an investor faces when trading with a dealer with bargaining power. (So $p_{t+1}^o\left(\varepsilon\right)$ is a nominal bid price for investors with

$\varepsilon < \varepsilon^*$, and the nominal ask price for investors with $\varepsilon > \varepsilon^*$.) With this notation, (46) can be written as

$$\Phi_{t+1} = \beta \mathbb{E}_t \kappa (v_{t+1}) (1-\theta) \left\{ G\left(\varepsilon^*_{t+1}\right) \mathcal{S}^b_{t+1} + \left[1 - G\left(\varepsilon^*_{t+1}\right)\right] \mathcal{S}^a_{t+1} \right\} \bar{\phi}_{t+1}.$$

Consider a dealer who has the bargaining power in a meeting with a random investor with preference type $\varepsilon < \varepsilon^*_{t+1}$. In equilibrium, this investor wishes to sell the A^s_{It+1} equity shares that he is holding to the dealer, who pays the investor $p^o_{t+1}(\varepsilon)$ dollars per share, therefore earning $p_{t+1} - p^o_{t+1}(\varepsilon)$ dollars per share, for a total intermediation profit equal to $\left[p_{t+1} - p^o_{t+1}(\varepsilon)\right] A^s_{It+1}$ dollars. Hence $\mathcal{S}^b_{t+1}$ is the expected value of the dealer's nominal profit from intermediation, conditional on having contacted an investor who wants to sell equity. Next consider a dealer who has the bargaining power in a meeting with a random investor with preference type $\varepsilon > \varepsilon^*_{t+1}$. In equilibrium, this investor wishes to use spend all his money holdings, A^m_{It+1}, to purchase shares from the dealer. The dealer charges the investor $p^o_{t+1}(\varepsilon)$ dollars per share, so at this price the investor wishes to buy $Q_{t+1}(\varepsilon)$ shares from the dealer. The dealer earns $p^o_{t+1}(\varepsilon) - p_{t+1}$ dollars per share in this transaction, for a total intermediation profit equal to $\left[p^o_{t+1}(\varepsilon) - p_{t+1}\right] Q_{t+1}(\varepsilon)$ dollars. Hence $\mathcal{S}^a_{t+1}$ is the expected value of the dealer's nominal profit from intermediation, conditional on having contacted an investor who wants to buy equity. Since the dealer can rebalance his portfolio of equity and cash freely in the interdealer market, the expected real value of his profit from intermediation is $\mathcal{S}^a_{t+1}\bar{\phi}_{t+1}$, conditional on having contacted a buyer, and $\mathcal{S}^b_{t+1}\bar{\phi}_{t+1}$ conditional on having met a seller. Hence $\beta \mathbb{E}_t \left\{ G\left(\varepsilon^*_{t+1}\right) \mathcal{S}^b_{t+1} + \left[1 - G\left(\varepsilon^*_{t+1}\right)\right] \mathcal{S}^a_{t+1} \right\} \bar{\phi}_{t+1}$ is the discounted expected real income from intermediation to a dealer in period $t+1$, conditional on his contacting an investor in the OTC market (with probability $\kappa(v_{t+1})$), and conditional on the dealer having the bargaining power in the bilateral negotiation with the investor (with probability $1-\theta$).

To simplify the exposition hereafter we specialize the analysis to the pure-dealer model with $\alpha = 0$. We also focus on stationary equilibria, that is, equilibria in which the measure of dealers and asset holdings are constant over time, i.e., $v_t = v$, $A^s_{Dt} = A^s_D$, and $A^s_{It} = A^s_I$, real asset prices are time-invariant functions of the aggregate dividend, i.e., $\phi^s_t = \phi^s y_t$, $p_t \phi^m_t \equiv \bar{\phi}^s_t = \bar{\phi}^s y_t$, $\phi^m_t A^m_{It} = Z y_t$, and $\phi^m_t A^m_{Dt} = Z_D y_t$. Hence, in a stationary equilibrium, $\varepsilon^*_t = \bar{\phi}^s - \phi^s \equiv \varepsilon^*$, $\phi^s_{t+1}/\phi^s_t = \bar{\phi}^s_{t+1}/\bar{\phi}^s_t = \gamma_{t+1}$, $\phi^m_t/\phi^m_{t+1} = \mu/\gamma_{t+1}$, and $p_{t+1}/p_t = \mu$, and $\Phi_t = \Phi$ for all t. We again let $\bar{\beta} \equiv \beta\bar{\gamma}$ and maintain the assumption $\mu > \bar{\beta}$, but the following proposition considers the limiting case $\mu \to \bar{\beta}$. A stationary equilibrium with entry is summarized by a sequence of nominal prices $\{\phi_t^m, p_t\}$, a sequence of real asset prices, $\left(\phi_t^s, \bar{\phi}_t^s\right)$, a pair of money holdings (A_D^m, A_I^m), a pair of equity holdings (A_D^s, A), a sequence of real balances, $\{Zy_t\}$, and a threshold ε^* that satisfy the conditions reported in Proposition 3 (with δ replaced by $\delta(v)$), together with a number of active dealers, v, that satisfies (47)

$$\bar{\Phi} - k \leq 0, \text{ with " = " if } v > 0, \tag{47}$$

where

$$\bar{\Phi} \equiv \bar{\beta}\kappa(v)(1-\theta)\left[A_I^s \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon) + Z \int_{\varepsilon^*}^{\varepsilon_H} \tfrac{\varepsilon - \varepsilon^*}{\varepsilon + \phi^s} dG(\varepsilon)\right]. \tag{48}$$

Proposition 15 *Assume* $k < \bar{\beta}(1-\theta)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s$. *The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as* $\mu \to \bar{\beta}$, *provided the bargaining power of dealers satisfies* $1 - \theta = 1 - \frac{-\kappa'(v)v}{\kappa(v)}$.

Proposition 15 establishes that with dealer entry, the Friedman rule achieves efficiency only if the Hosios (1990) condition is satisfied.[17] Under the Hosios condition, the share of the gain from trade that is apportioned by dealers in bilateral meetings, $1-\theta$, equals the elasticity of the aggregate number of meetings with respect to the number of participating dealers, $1 + \kappa'(v) v / \kappa(v)$ (i.e., the contribution that the marginal dealer makes to the matching process). Thus generically, deviations from the Friedman rule could be welfare enhancing in the absence of other policies designed to restore the efficiency of the dealers' entry decision.

8 The Fed Model and the Modigliani-Cohn hypothesis

The high inflation of the 1970s stimulated researchers to ask whether stocks are a good hedge against inflation. In a well-known paper, Fama and Schwert (1977) found that, contrary to long-held beliefs, common stocks were rather perverse as hedges against inflation. They found that common stock returns were negatively related to the expected inflation rate during the 1953-71 period, and that they also seemed to be negatively related to the unexpected inflation rate.[18] In line with these observations, in the late 1970s Modigliani and Cohn (1979) pointed out that the ratio of market value to profits of firms had declined consistently since the late

[17] This is a standard result in the monetary search literature, see, e.g., Berentsen et al. (2007).

[18] At the time of Fama and Schwert's writing, Lintner (1975), Jaffe and Mandelker (1976), Bodie (1976), and Nelson (1976) had offered similar empirical evidence. Cagan (1974) is an early effort to study these issues using some historical records.

1960s. They observed that this fact was consistent with investors who capitalize equity earnings using a nominal interest rate instead of a real one, and settled on this kind of money illusion as the most reasonable explanation. More recently, Sharpe (2002), Asness (2000), Lansing (2004) and many others have documented that yields on stocks (e.g., as measured by the dividend-price ratio) are highly correlated with nominal bond yields.[19] Since stocks are claims to cash flows from real capital and inflation is the main driver of nominal interest rates, this correlation has proven difficult to rationalize with conventional asset pricing theory.[20]

Theory aside, this correlation has lead financial practitioners to adopt the so-called *Fed Model* of equity valuation to calculate the "correct" price of stocks.[21] In its simplest form, the Fed Model says that, because stocks and nominal Treasury bonds compete for space in investors' portfolios, their yields should be positively correlated. That is, if the yield on bonds rises, then the yield on stocks must also rise to maintain the competitiveness of stocks vis a vis bonds. Practitioners use this reasoning to argue that the yield on nominal bonds (plus a risk premium to account for the relative riskiness of stocks) defines a "normal" yield on stocks: if the measured stock yield is below this normal yield, then stocks are considered overpriced; if the measured stock yield is above this normal yield, then stocks are considered underpriced.[22]

The relationship between equity prices and monetary policy also appears to have been clear in the minds of policymakers. Alan Greenspan, for example, famously held the view that stock market booms are more likely to occur when inflation is low. He saw a dilemma in the use of monetary policy to defuse stock market booms:

[19] Along similar lines, Bordo and Wheelock (2007) review the histories of major 20th century stock market booms in the United States and nine other countries. They find that booms usually arose when inflation was below its long-run average, and that booms typically ended when inflation began to rise and/or monetary authorities tightened policy in response to rising or a threatened rise in inflation. Ritter and Warr (2002) argue that the decline in inflation was a major factor leading to the bull market of 1982-1999.

[20] See the discussions in Ritter and Warr (2002), Asness (2003), and Campbell and Vuolteenaho (2004).

[21] Sharpe (2002), Asness (2003), and Feinman (2005) discuss the popularity of the Fed Model among Wall Street analysts and strategists.

[22] The term *Fed Model* appears to have been first used by securities strategist Ed Yardeni in 1997 following the publication of the Federal Reserve Humphrey-Hawkins Report for July 1997. In Section 2 ("Economic and Financial Developments in 1997"), a chart plotted the time series for the earnings-price ratio of the S&P 500 against the 10-year constant-maturity nominal treasury yield and reported: "The run-up in stock prices in the spring was bolstered by unexpectedly strong corporate profits for the first quarter. Still, the ratio of prices in the S&P 500 to consensus estimates of earnings over the coming twelve months has risen further from levels that were already unusually high. Changes in this ratio have often been inversely related to changes in long-term Treasury yields, but this year's stock price gains were not matched by a significant net decline in interest rates. As a result, the yield on ten-year Treasury notes now exceeds the ratio of twelve-month-ahead earnings to prices by the largest amount since 1991, when earnings were depressed by the economic slowdown."

"We have very great difficulty in monetary policy when we confront stock market bubbles. That is because, to the extent that we are successful in keeping product price inflation down, history tells us that price-earnings ratios under those conditions go through the roof. What is really needed to keep stock market bubbles from occurring is a lot of product price inflation, which historically has tended to undercut stock markets almost everywhere. There is a clear trade-off. If monetary policy succeeds in one, it fails in the other. Now, unless we have the capability of playing in between and managing to know exactly when to push a little here and to pull a little there, it is not obvious to me that there is a simple set of monetary policy solutions that deflate the bubble." (Alan Greenspan, FOMC transcript, September 24, 1996, pp. 30-31.)

To fix ideas, consider a standard Lucas (1978) economy in which a risk-neutral investor with discount rate β prices a tree that is subject to a shock that renders it permanently unproductive with probability $1-\pi$. Conditional on remaining productive, the tree yields real dividend D_t, with $D_{t+1}=\gamma_{t+1}D_t$, where γ_{t+1} is a nonnegative random variable with mean $\bar{\gamma}\in(0,(\beta\pi)^{-1})$. The real price of an equity share of the tree is

$$P_t^s=\frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}D_t.$$

If we set $D_t=\bar{\varepsilon}y_t$, this is just the equity price in the nonmonetary equilibrium of Proposition 3. We can use this expression to obtain

$$\frac{\bar{D}_{t+1}}{P_t^s}=(1+r)-\bar{\gamma}\pi, \tag{49}$$

where $\bar{D}_{t+1}\equiv\bar{\gamma}\pi D_t$ denotes the expected dividend (conditional only on the tree having survived period t) and $1+r=1/\beta$ is the real risk-free rate. Condition (49) is known as the "Gordon growth model" (e.g., Gordon (1962). Williams (1938)). The left side is the *dividend (or stock) yield*, which is equal to the real risk-free rate, $1+r$, minus the expected growth rate of the real dividend, $\bar{D}_{t+1}/D_t=\bar{\gamma}\pi$. All the variables in (49) are real. In particular, since a rational investor's Euler equation equates the expected real equity return to the risk-free real interest rate, i.e., $\mathbb{E}_t R_{t+1}^s=1+r$, where $R_{t+1}^s\equiv\pi\frac{(P_{t+1}^s+y_{t+1})}{P_t^s}$, the investor uses the real interest rate r to discount future dividends.[23] According to the narrative behind the Fed Model, however,

[23] As before, $\mathbb{E}_t$ is the expectation over y_{t+1}, conditional on the information available in period t, as well as on the tree surviving period $t+1$.

investors allocate their portfolio between stocks and nominal long-term bonds by comparing the expected real return on equity to the expected *nominal* bond yield, i.e., they use the wrong Euler equation, $\mathbb{E}_t R_{t+1}^s = 1 + \iota$. This leads to

$$\frac{\bar{D}_{t+1}}{P_t^s} = (1 + \iota) - \bar{\gamma}\pi,$$

which "explains" the positive relation between the nominal bond yield $\iota = (\mu - \bar{\beta})/\bar{\beta}$ and the stock yield $\bar{D}_{t+1}/P_t^s$ by saying that investors suffer from a money illusion in the sense that they discount future real dividends using the nominal rate, ι, rather than the real rate, r. This is the Modigliani-Cohn hypothesis. Financial analysts are often ambivalent toward the Fed Model. On the one hand, it "works." On the other hand, they are reluctant to recommend it to clients because the conventional logic behind it is fundamentally flawed; it is inconsistent with investor rationality.[24]

Despite much skepticism, however, the Modigliani-Cohn hypothesis remains the leading explanation for the positive correlation between stock yields and nominal bond yields, and for the empirical success of the Fed Model. Campbell and Vuolteenaho (2004) empirically decompose the S&P500 stock yield into three components: (*i*) a rational forecast of long-run expected dividend growth, (*ii*) a subjective risk premium (identified from a cross-sectional regression), and (*iii*) a residual "mispricing term." They evaluate three hypotheses for why low stock prices coincide with high inflation: (1) High inflation coincides with low expected

[24]For example, Feinman (2005) (Chief Economist at Deutsche Management Americas in New York) writes: "If interest rates are to be brought into the calculus at all, they should be real rates, not nominal. This is not to deny that equity prices seem to be set as if investors are comparing equity yields with nominal interest rates. But this just demonstrates the error of money illusion. It should not be construed as recommending that error." Similarly, Asness (2003) writes: "Historically S&P 500 earnings-price ratio and 10-Year Treasury Rate have been strongly related... The correlation of these two series over this period [1965-2001] is an impressive +0.81. I am far from unique in presenting a graph like figure 1. It's a rare Wall Street strategist that in the course of justifying the Fed Model (or similar analytic) does not pull out a version of this figure... If you are trying to explain why price-earning ratios are where they are, based on investors behaving in a similar manner in the past (errors and all), then feel free to use the Fed Model (hopefully modified for volatility as in this paper), but do not confuse that with a tool for making long-term recommendations to investors." Siegel (2002): "It is true that bonds are the major asset class that competes with stocks in an investor's portfolio, so one might expect that low interest rates would be favorable to stocks. But since in the long run low interest rates are caused by low inflation, the rate of growth of nominal earnings, which depends in large part on the rate of inflation, will be lower also. Over long periods of time, changes in the inflation rate cause changes in earnings growth of the same magnitude and do not change the valuation of stocks." See also Ritter and Warr (2002). Modigliani and Cohn (1979) themselves expressed some skepticism about their money illusion hypothesis: "...we readily admit that our conclusion is indeed hard to swallow—and especially hard for those of us who have been preaching the gospel of efficient markets. It is hard to accept the hypothesis of a long-lasting, systematic mistake in a well-organized market manned by a large force of alert and knowledgeable people."

dividend growth. (2) High inflation coincides with a high (subjective) risk premium. (3) Investors suffer from money illusion. Campbell and Vuolteenaho (2004) assess these hypotheses by regressing the three components of the dividend yield (the expected dividend growth, risk premium, and the residual mispricing term) on an exponentially smoothed moving average of inflation. The regression coefficient of expected dividend growth on inflation is positive and large, so the raw correlation between inflation and expected dividend growth is not negative as required by the first hypothesis. The regression coefficient of the risk premium on inflation is negative but small, indicating that the risk premium is not increasing with inflation as required by the second hypothesis. Thus, Campbell and Vuolteenaho (2004) reject the two conventional rational hypotheses for the positive correlation between the dividend yield and inflation. The regression coefficient of the residual mispricing term on inflation is positive, large, and statistically significant. Moreover, the R^2 on this regression is 77.90, indicating that inflation accounts for about 80% of the variability in the mispricing term. Based on this evidence, Campbell and Vuolteenaho (2004) conclude that the positive correlation of the dividend yield with inflation is mostly due to the mispricing term, i.e., stocks appear to be undervalued by conventional measures when inflation is high.[25] In the remainder of this section we show our theory offers a novel explanation for this finding: the effects that inflation has on asset prices through the liquidity (or resalability) channel. The new explanation we propose will be transparent because our theory does not assume irrational investors that suffer money

[25] To the extent that these types of studies do not fully control for risk, the results may confound the impact of risk attitudes and label attribute them to some anomaly such as money illusion. Cohen et al. (2005) revisit the robustness of the results of Campbell and Vuolteenaho (2004) by further controlling for changes in the risk premium. They exploit the fact that if the equity premium is high for risk-related reasons, then there is a cross-sectional implication, namely that high-beta stocks should outperform low-beta stocks in such periods. The Modigliani-Cohn hypothesis, on the other hand, implies that inflation-driven mispricing will apply to all stocks equally, causing all stocks to be equally underpriced when inflation is high. Cohen et al. (2005) show the latter is the case and interpret this as further confirmation of the Modigliani-Cohn hypothesis. Bekaert and Engstrom (2010) find the bulk of the contribution to the covariance between equity and bond yields comes from the positive comovements between expected inflation and a residual term, just like Campbell and Vuolteenaho (2004). However, Bekaert and Engstrom (2010) claim that this is due to a correlation between expected inflation and two plausible proxies for rational time-varying risk premia: a measure of economic uncertainty (the uncertainty among professional forecasters regarding real GDP growth) and a consumption-based measure of risk aversion. Thus, they offer a rational channel that could potentially explain why the Fed model "works:" their explanation is that high expected inflation coincides with periods of high risk aversion and/or economic uncertainty, which conflicts with Campbell and Vuolteenaho (2004) and Cohen et al. (2005). The papers differ on the specifics of how they measure equity risk premia and on the sample period (Bekaert and Engstrom focus on the post-war subsample while Campbell and Vuolteenaho and Cohen et al. go back to 1930s). Bekaert and Engstrom (2010) conclude that "Using this data set alone, it is likely hard to definitively exclude the money illusion story in favor of our story."

illusion and it abstracts from the other channels through which high inflation may depress real asset prices, such as the possibility that it may adversely affect firms profitability or riskiness.

Consider the pure-dealer version of the model analyzed in Proposition 3 (the main idea would not change if we consider the nonintermediated market structure of Proposition 4). The equilibrium equity price is

$$\phi_t^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\epsilon(\iota)\, y_t$$

where $\epsilon(\iota) \equiv \max\{\varepsilon^*, \bar{\varepsilon}+\delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon\}$. Since ε^* is decreasing in the growth rate of the money supply, μ, and the nominal bond yield, ι, is monotonic in μ, we have $\epsilon'(\iota) < 0$. Let $\bar{y}_{t+1} \equiv \bar{\gamma}\pi y_t$ denote the expected dividend (conditional only on the tree having survived period t). The log dividend yield is

$$\log \bar{y}_{t+1} - \log \phi_t^s = \log[(1+r) - \bar{\gamma}\pi] - \log \epsilon(\iota), \tag{50}$$

and it is increasing in the nominal yield, ι. Thus, (50) rationalizes the Fed Model, despite the fact that agents do not suffer from money illusion (they discount payoffs using the risk-free real rate $1+r$), risk premia do not change (since agents are risk-neutral here), and the expected growth rate of the dividend, $\bar{\gamma}\pi$, is unaffected by monetary considerations.

9 Liquidity crises and market crashes

In this section we show that the model of Section 7 admits dynamic rational-expectations equilibria that exhibit episodes that resemble "liquidity crises" and "market crashes," and we use the model to explore the scope for welfare enhancing monetary interventions. The basic idea is to construct equilibria with random belief-driven oscillations in asset prices and standard measures of market liquidity, such as trade volume, spreads, execution delays borne by investors, and liquidity provision by dealers (both by matching buyers and sellers as well as by holding asset inventories on their own account). To this end, we construct stationary sunspot equilibria in the model with a pure-dealer OTC market and endogenous entry of dealers.[26]

[26] Our sunspot equilibria are of course in the spirit of Cass and Shell (1983). Such equilibria are quite common in monetary models: Lagos and Wright (2003) study dynamic equilibria in the baseline Lagos and Wright (2005) model and find a rich set of dynamic equilibria, including cycles and sunspot equilibria. Rocheteau and Wright (2013) find sunspot equilibria in a nonmonetary version of Lagos and Wright (2005) in which consumers use equity shares on a Lucas tree to pay for consumption goods, e.g., as in Lagos (2010), and in which there is free entry of producers. In the context of the search-based literature on OTC financial markets, dynamic equilibria are studied in Lagos and Rocheteau (2009) and Lagos et al. (2011), where equilibrium is unique and converges monotonically to the unique steady state. Lagos and Rocheteau (2009) find that for some parametrizations, a

Suppose that in the second subperiod of period t, all agents observe the realization $X_{t+1} \in \mathbb{X}$ of an extraneous random variable, a "sunspot" that evolves according to a Markov chain, with $\sigma_{ij} = \Pr(X_{t+1} = j | X_t = i) \in (0,1)$, where $\sum_{j \in \mathbb{X}} \sigma_{ij} = 1$ for each $i \in \mathbb{X}$. It can be shown that in this formulation the marginal investor type in the OTC market is $\varepsilon_t^* = \frac{p_t \tilde{\phi}_t^m - \tilde{\phi}_t^s}{y_t}$, where $\tilde{\phi}_t^k \equiv \mathbb{E}_{X_{t+1}} \left[\phi_t^k | X_t \right]$, for $k = m, s$. We explore the existence of equilibria in which allocations and prices are time invariant functions of the aggregate dividend and the sunspot. That is, equilibria in which for all t such that $X_{t+1} = j \in \mathbb{X}$, $v_{t+1} = v_j$, $A_{Dt+1}^s = A_{Dj}^s$, and $A_{It+1}^s = A_{Ij}^s$, with $\phi_t^s = \phi_j^s y_t$, $p_t \phi_t^m \equiv \phi_t^o = \phi_j^o y_t$, $\phi_t^m A_{It}^m = \phi_t^m A_t^m = Z_j y_t$, and therefore $\varepsilon_{t+1}^* = \tilde{\phi}_j^o - \tilde{\phi}_j^s \equiv \varepsilon_j^*$, where $\tilde{\phi}_j^o \equiv \sum_{i \in \mathbb{X}} \sigma_{ji} \phi_i^o$ and $\tilde{\phi}_j^s \equiv \sum_{i \in \mathbb{X}} \sigma_{ji} \phi_i^s$. (As usual, dealers never hold money overnight.) The Euler equations for real balances and equity between any two periods t and $t+1$ with $X_{t+1} = i$ are

$$Z_i = \frac{\bar{\beta}}{\mu} \left[1 + \delta(v_i)\,\theta \int_{\varepsilon_i^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon_i^*}{\varepsilon_i^* + \tilde{\phi}_i^s} dG(\varepsilon) \right] \tilde{Z}_i \tag{51}$$

$$\phi_i^s = \bar{\beta}\pi \left[\tilde{\phi}_i^s + \max\left(\varepsilon_i^*, \bar{\varepsilon} + \delta(v_i)\,\theta \int_{\varepsilon_L}^{\varepsilon_i^*} G(\varepsilon)\, d\varepsilon \right) \right], \tag{52}$$

where $\tilde{Z}_i \equiv \sum_{j \in \mathbb{X}} \sigma_{ij} Z_j$. A stationary sunspot equilibrium is a vector $(v_i, \tilde{Z}_i, \tilde{\phi}_i^s, A_{Di}^s, A_{Ii}^s, \varepsilon_i^*)_{i \in \mathbb{X}}$ that satisfies

$$\tilde{Z}_i = \frac{\bar{\beta}}{\mu} \sum_{j \in \mathbb{X}} \sigma_{ij} \left[1 + \delta(v_j)\,\theta \int_{\varepsilon_j^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon_j^*}{\varepsilon_j^* + \tilde{\phi}_j^s} dG(\varepsilon) \right] \tilde{Z}_j \tag{53}$$

$$\tilde{\phi}_i^s = \bar{\beta}\pi \sum_{j \in \mathbb{X}} \sigma_{ij} \left[\tilde{\phi}_j^s + \max\left(\varepsilon_j^*, \bar{\varepsilon} + \delta(v_j)\,\theta \int_{\varepsilon_L}^{\varepsilon_j^*} G(\varepsilon)\, d\varepsilon \right) \right], \tag{54}$$

with

$$\tilde{Z}_j = \frac{A_{Dj}^s + \delta(v_j)\, G(\varepsilon_j^*) A_{Ij}^s}{\delta(v_j)\,\theta[1 - G(\varepsilon_j^*)] \frac{1}{\varepsilon_j^* + \tilde{\phi}_j^s} + \delta(v_j)(1-\theta) \int_{\varepsilon_j^*}^{\varepsilon_H} \frac{1}{\varepsilon + \tilde{\phi}_j^s} dG(\varepsilon)} \tag{55}$$

$$k = \bar{\beta}(1-\theta) \frac{\delta(v_j)}{v_j} \left[A_{Ij}^s \int_{\varepsilon_L}^{\varepsilon_j^*} (\varepsilon_j^* - \varepsilon) dG(\varepsilon) + \tilde{Z}_j \int_{\varepsilon_j^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon_j^*}{\varepsilon + \tilde{\phi}_j^s} dG(\varepsilon) \right] \tag{56}$$

$$A_{Ij}^s = \begin{cases} (1-\pi) A^s & \text{if } \bar{\varepsilon} + \delta(v_j)\,\theta \int_{\varepsilon_L}^{\varepsilon_j^*} G(\varepsilon)\, d\varepsilon \le \varepsilon_j^* \\ A^s & \text{if } \varepsilon_j^* < \bar{\varepsilon} + \delta(v_j)\,\theta \int_{\varepsilon_L}^{\varepsilon_j^*} G(\varepsilon)\, d\varepsilon \end{cases} \tag{57}$$

$$A_{Dj}^s = A^s - A_{Ij}^s. \tag{58}$$

version of their model in which the number of dealers is determined endogenously by an entry condition delivers multiple steady states.

$\beta = (0.99)^{1/365}$	$\bar{\gamma} = E\left(\frac{y_{t+1}}{y_t}\right) = (1.04)^{1/365}$
$\varepsilon \sim U\,[0.01, 20]$	$\Sigma = SD\left(\frac{y_{t+1}-y_t}{y_t}\right) = \frac{0.12}{\sqrt{365}}$
$\delta(v) = 1 - e^{-(0.1)v}$	$\pi = (0.9)^{1/365}$
$k = 0.1$	$\theta = 0.5$
$y_{t+1} = \bar{\mu} e^{x_{t+1}} y_t$	$\mu = (1.03)^{1/365}$
$x_{t+1} \sim \mathcal{N}\left(-\Sigma^2/2, \Sigma^2\right)$	$\sigma_{00} = (0.996)^{1/365}$; $\sigma_{11} \approx 1$

Table 1: Parametrization for stationary sunspot equilibrium

Conditions (53) and (54) are obtained from (51) and (52) by multiplying the latter through by σ_{ji}, summing over all $i \in \mathbb{X}$, and relabeling the subindices.[27] For any t and $t+1$, suppose that $X_t = i$ and $X_{t+1} = j$, then the equilibrium growth rates of asset prices are

$$\frac{\phi^s_{t+1}}{\phi^s_t} = \frac{\phi^s_j}{\phi^s_i}\gamma_{t+1}, \frac{\bar{\phi}^s_{t+1}}{\bar{\phi}^s_t} = \frac{\bar{\phi}^s_j}{\bar{\phi}^s_i}\gamma_{t+1}, \frac{\phi^m_t}{\phi^m_{t+1}} = \frac{Z_i}{Z_j}\frac{\mu}{\gamma_{t+1}}, \text{ and } \frac{p_{t+1}}{p_t} = \frac{Z_i\bar{\phi}^s_j}{Z_j\bar{\phi}^s_i}\mu.$$

That is, asset prices follow the dynamic paths implied by the growth rates of the dividend and the money supply, but the paths themselves are shifted by sunspots.

Although the economy is admittedly stylized, we nonetheless calibrate the critical parameters in order to explore the ability of the theory to generate episodes that resemble liquidity crises for reasonable parametrizations. We think of a model period as being a day. The discount factor, β, is chosen so that the annual real risk-free rate equals 1%. The dividend growth rate is independently lognormally distributed over time, with mean 1.04 and standard deviation 0.12 (per annum), e.g., as in Lettau and Ludvigson (2005). We choose π so that a tree has a 90% chance of remaining productive each year. The dealer's bargaining power is set to 1/2. When the measure of active dealers is v, the probability that an investor contacts a dealer in the OTC market is $\delta(v) = 1 - e^{-(0.1)v}$. The entry cost for dealers is 1% of the average valuation of the aggregate dividend, i.e., $k = 0.1$. The growth rate of the money supply is 3% per year. Finally, we let $\mathbb{X} = \{0, 1\}$, and identify $i = 0$ with "normal times" and $j = 1$ with the "liquidity-crisis state." The sunspot is a rare event: both σ_{00} and σ_{11} are chosen very close to 1. The parametrization is summarized in Table 1.

Table 2 reports the key equilibrium variables during normal times and in the liquidity-crisis state. Figure 3 displays a typical equilibrium path of our daily model simulated for 100 years.

[27]It is convenient to define sunspot equilibrium in terms of the conditional expectations of the equity prices, $(\tilde{Z}_i, \tilde{\phi}^s_i)_{i\in\mathbb{X}}$, rather than the realized equity prices, $(Z_i, \phi^s_i)_{i\in\mathbb{X}}$, but given the former, the latter are immediate from (51) and (52).

ϕ_0^s/ϕ_1^s	$\delta(v_0)$	$\delta(v_1)$	Z_0/Z_1	$\varepsilon_0^*/\varepsilon_1^*$	A_{D0}^s	A_{D1}^s
1.17	0.87	0.04	12.9	2.90	1	0

Table 2: Stationary sunspot equilibrium

The market starts in the normal state, and the sunspot switches to the liquidity-crisis state in the first day of year 50. On that day the stock market falls by 17%, as illustrated by the top left panel. (The lighter path shown in the top left, bottom middle, and bottom right panels is the path the variable would have followed if the sunspot had not switched.) On the same day, dealers withdraw from market making and this causes the daily trading probability for an investor to fall from 90% to 4%, as illustrated by the top middle panel. The right panel in the top row shows that dealer's also stop supplying liquidity by carrying assets overnight, which exacerbates the misallocation of the asset. In the bottom row, the left panel shows that the average real spread per dollar traded, $\bar{\mathcal{S}}_t$, increases drastically, and the middle panel shows that trade volume collapses abruptly when the crisis hits. The right panel in the bottom row shows the behavior of real balances. In sum, Figure 3 shows time series with the hallmarks of a liquidity crisis: a sharp sudden decline in marketmaking and trade volume accompanied by a sharp sudden increase in trading delays and spreads, at the same time that there is a sharp sudden crash in asset prices. In layman words: *liquidity dries up* and the *bubble bursts*.

The rationale behind the sunspot equilibrium goes as follows. Dealers withdraw from market making because they expect low profit from the intermediation business. This makes trading more difficult for investors (trading delays increase), so real balances, which investors use as means of payment in financial transactions, fall. As a result, investors with high asset valuations cannot afford to buy as much equity from low valuation investors. This causes the equity price to crash, and in turn, lower equity prices, lower trading activity, and lower real balances all validate the dealer's decision to withdraw from markets. The self-referential nature of market liquidity is critical, both in terms of the ability of investors to find counterparties for trade, as well as the endogenous value of the means of payment (in this case fiat money).

10 Related literature

The model builds on two strands of literature: the Search Theory of Money, and search-based models of financial trade in OTC markets. Specifically, we embed an OTC financial trading arrangement similar to Duffie et al. (2005) into a Lagos and Wright (2005) economy.

In the standard formulations of the Lagos-Wright framework, money (and sometimes other assets) are used as payment instruments to purchase consumption goods in bilateral markets mediated by search. We instead posit that money is used as a medium of exchange in OTC markets for financial assets. In the standard monetary model, money and other liquid assets help to allocate goods from producers to consumers, while in our current formulation, money helps to allocate financial assets among traders with heterogeneous valuations. This shift in the nature of the gains from trade offers a different perspective that delivers novel insights on the interaction between monetary policy and financial markets.

As a model of financial trade, the main strength of Duffie et al. (2005) is perhaps its realistic OTC market structure consisting of an interdealer market and bilateral negotiated trades between investors, and between investors and dealers. In Duffie et al. (2005), agents who wish to buy assets pay sellers with linear-utility transfers. In addition, utility transfers from buyers to sellers are unconstrained, so buyers effectively face no budget constraints in financial transactions. Our formulation keeps the appealing market structure of Duffie et al. (2005) but improves upon its stylized model of financial transactions by considering traders who face standard budget constraints and use fiat money to purchase assets. These modifications make the standard OTC formulation amenable to general equilibrium analysis, and deliver a natural transmission mechanism through which monetary policy influences financial markets.

Our work is related to previous studies, e.g., Geromichalos et al. (2007), Jacquet and Tan (2010), Lagos and Rocheteau (2008), Lagos (2010a, 2010b, 2011), Lester et al. (2012), Nosal and Rocheteau (2013), that introduce a real asset that can (at least to some degree) be used along with money as a medium of exchange for consumption goods in variants of Lagos and Wright (2005). These papers identify the liquidity value of the asset with its usefulness in exchange, and find that when the asset is valuable as a medium of exchange, this manifests itself as a "liquidity premium" that makes the real asset price higher than the expected present discounted value of its financial dividend. High anticipated inflation reduces real money balances; this tightens bilateral trading constraints, which in turn increases the liquidity value and the real price of the asset. In contrast, we find that real asset prices are decreasing in the rate of anticipated inflation. There are some models that also build on Lagos and Wright (2005) where agents can use a real asset as collateral to borrow money that they subsequently use to purchase consumption goods. In those models, anticipated inflation reduces the demand for real balances which can in turn reduce the real price of the collateral asset needed to borrow money (see, e.g., He et al., 2012, and Li and Li, 2012). The difference is that in our setup inflation reduces the real asset price by constraining the reallocation of the financial asset from investors with low valuations to investors with relatively high valuations.

We share with two recent papers, Geromichalos and Herrenbrueck (2012) and Trejos and Wright (2012), the interest in bringing models of OTC trade in financial markets within the realm of modern monetary general equilibrium theory. Trejos and Wright (2012) offer an in-depth analysis of a model that nests Duffie et al. (2005) and the prototypical "second generation" monetary search model with divisible goods, indivisible money and unit upper bound on individual money holdings (e.g., Shi, 1995 or Trejos and Wright, 1995). Trejos and Wright emphasize the different nature of the gains from trade in both classes of models. In monetary models agents value consumption goods differently and use assets to buy goods, while in Duffie et al. (2005) agents trade because they value assets differently, and goods which are valued the same by all investors are used to pay for asset purchases. In our formulation there are gains from trading assets, as in Duffie et al. (2005), but agents pay with money, as in standard monetary models. Another difference with Trejos and Wright (2012) is that rather than assuming indivisible assets and unit upper bound on individual asset holdings as in Shi (1995), Trejos and Wright (1995) and Duffie et al. (2005), we work with divisible assets and unrestricted portfolios, as in Lagos and Wright (2005) and Lagos and Rocheteau (2009).

Geromichalos and Herrenbrueck (2012) is methodologically closer to our work. They extend Lagos and Wright (2005) by incorporating a real asset that by assumption cannot be used to purchase goods in the decentralized market (as usual, at the end of every period agents choose next-period money and asset portfolios in a centralized market). The twist is that at the very beginning of every period, agents learn whether they will want to buy or sell consumption goods in the subsequent decentralized market and at that point they have access to a bilateral search market where they can retrade money and assets. This market allows agents to rebalance their positions depending on their need for money, e.g., those who will be buyers seek to buy money and sell assets. So although assets cannot be directly used to purchase consumption goods as in Geromichalos et al. (2007) or Lagos and Rocheteau (2008), agents can use assets to buy goods indirectly, i.e., by exchanging them for cash in the additional bilateral trading round at the beginning of the period. Geromichalos and Herrenbrueck use the model to revisit the link between asset prices and inflation. Their core results (they have several others) are similar to those obtained in models where the asset can be used directly as a medium of exchange for consumption goods, i.e., the asset carries a liquidity premium and higher inflation increases the real asset price in the centralized market. There are relevant differences between our work and Geromichalos and Herrenbrueck (2012). In our setup money allows agents to exploit gains from trading assets (as in Duffie et al.) rather than consumption goods (as in the money literature), which is why we instead find that inflation reduces asset prices. Also, we consider an OTC market with dealers who act as intermediaries, which allows us to study the effect of monetary policy on bid-ask spreads and dealers' incentives to supply liquidity services—the dimensions of financial liquidity that search based theories of OTC markets seek to explain.

The fact that the equilibrium asset price is larger than the expected present discounted value that any agent assigns to the dividend stream is reminiscent of the literature on speculative trading that can be traced back to Harrison and Kreps (1978). As in Harrison and Kreps, in our model speculation arises because traders have heterogeneous asset valuations that change over time: investors are willing to pay for the asset more than the present discounted value that they assign to the dividend stream, in anticipation of the capital gain they expect to obtain when reselling the asset to higher-valuation investors in the future. In terms of differences, in Harrison and Kreps traders have heterogeneous stubborn beliefs about the stochastic dividend process, and their motive for trading is that they all believe (at least some of them mistakenly) that by trading the asset they can profit at the expense of others. In our formulation traders simply have stochastic heterogeneous valuations for the dividend, as in Duffie et al. (2005). Our model offers a new angle on the speculative premium embedded in the asset price, by showing how it depends on the underlying financial market structure and the prevailing monetary policy that jointly determine the likelihood and profitability of future resale opportunities.

11 Conclusion

We have developed a model in which money is used as a medium of exchange in financial transactions that take place in over-the-counter markets. In any monetary equilibrium the real asset price contains a speculative premium that is positively related to the quantity of real money balances and therefore negatively correlated with anticipated inflation and the long-term nominal interest rate. As a result, the asset price generically exceeds the expected present discounted value that any agent assigns to the dividend stream. We have shown that this simple mechanism rationalizes the positive correlation between the real yield on stocks and the nominal yield on Treasury bonds—an empirical observation long regarded anomalous. We have also used the model to study how monetary considerations and the microstructure where the asset is traded jointly determine the standard measures of financial liquidity of OTC markets, such as the size of bid-ask spreads, the volume of trade, and the incentives of dealers to supply immediacy, both by choosing to participate in the market-making activity, as well as by holding asset inventories on their own account. We have shown that there exist multiple equilibria as well as dynamic equilibria that resemble expectation driven "market crashes" or "liquidity crises" in which market liquidity suddenly dries up: dealers drastically reduce their market-making activity, trade volume drops, bid-ask spreads widen, and asset prices fall abruptly.

We conclude by mentioning what we think are two promising avenues for future work. First, given that the model can generate inefficient liquidity crises, it would be interesting to explore the scope for welfare enhancing monetary policy, both conventional, e.g., by changing inflation and nominal rates, and unconventional, e.g., by issuing fiat money to purchase assets whose markets have suddenly become illiquid. Second, the model could be useful to interpret the behavior of asset prices in OTC markets. Recently Ang et al. (2013) have analyzed a large cross section of OTC-traded common stocks from 1977 through 2008 and find that equity returns are increasing in the proportion of non-trading days (i.e., days in which the stock was not traded) and decreasing in the trade volume of the stock. They interpret these findings through the lens of asset pricing theories that emphasize differences in investors' individual valuations (e.g., due to differences in opinions about the fundamentals) and limits on short sales. Our theory also has heterogeneous valuations, but in addition, it is explicit about the search and bargaining frictions that are defining characteristics of OTC markets. It is also consistent with the behavior of the illiquidity premia in response to variations in the measures of liquidity documented by Ang et al. (2013): In the stationary monetary equilibrium the expected financial return on the equity, $(\phi_{t+1}^s + y_{t+1})/\phi_t^s$, is decreasing with δ (see Proposition 7). The theory we have developed also has sharp implications about how the effect of monetary policy on asset prices depends on the microstructure of the market. For instance, it predicts that the speculative premium, and therefore the typical residual mispricing term, should be larger and more responsive to inflation in markets that are more liquid from an investor standpoint, i.e., markets where investors trade fast and face narrow spreads.

A Proofs

Proof of Proposition 1. The choice variable a'_{tD} does not appear in the Planner's objective function, so $a'_{tD} = 0$ at an optimum. Also, (5) must bind for every t at an optimum, so the planner's problem is equivalent to

$$\max_{\{\tilde{a}_{tD},\tilde{a}_{tI},\underline{a}_{tib(i)},a'_{tI}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\delta \int_{[\varepsilon_L,\varepsilon_H]} \varepsilon a'_{tI}(d\varepsilon) + (1-\alpha-\delta)\bar{\varepsilon} a_{tI} \right.$$

$$\left. + \int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \le b(i)\}} \left[\varepsilon_i \underline{a}_{tib(i)} \left(\varepsilon_i, \varepsilon_{b(i)}\right) + \varepsilon_{b(i)} \underline{a}_{tb(i)i} \left(\varepsilon_{b(i)}, \varepsilon_i\right) \right] dG(\varepsilon_i)\, dG\left(\varepsilon_{b(i)}\right) di \right] y_t$$

$$\text{s.t. (2), (3), (6), (7) and } \delta \int_{[\varepsilon_L,\varepsilon_H]} a'_{tI}(d\varepsilon) \le v a_{tD} + \delta a_{tI}.$$

Let W^* denote the maximum value of this problem. Then clearly, $W^* \le \bar{W}^*$, where

$$\bar{W}^* = \max_{\{\tilde{a}_{tD},\tilde{a}_{tI}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \le b(i)\}} \max\left(\varepsilon, \varepsilon'\right) 2\tilde{a}_{tI} dG(\varepsilon)\, dG\left(\varepsilon'\right) di \right.\right.$$

$$\left.\left. + \varepsilon_H \left(v \tilde{a}_{tD} + \delta \tilde{a}_{tI}\right) + (1-\alpha-\delta)\bar{\varepsilon}\tilde{a}_{tI} \right] \pi y_t \right] + w,$$

s.t. (3), where $w \equiv [\alpha\varepsilon_B + \delta\varepsilon_H + (1-\alpha-\delta)\bar{\varepsilon}](1-\pi) A^s \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t\right)$ and

$$\varepsilon_B \equiv \int \int \max\left(\varepsilon, \varepsilon'\right) dG(\varepsilon)\, dG\left(\varepsilon'\right).$$

Rearrange the expression for $\bar{W}^*$ and substitute (3) (at equality) to obtain

$$\bar{W}^* = \max_{\{\tilde{a}_{tI}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_H A^s + \left[\alpha\varepsilon_B + \delta\varepsilon_H + (1-\alpha-\delta)\bar{\varepsilon} - \varepsilon_H\right] \tilde{a}_{tI} \right\} \pi y_t \right\} + w$$

$$= \left\{ \pi\varepsilon_H + (1-\pi)\left[\alpha\varepsilon_B + \delta\varepsilon_H + (1-\alpha-\delta)\bar{\varepsilon}\right] \right\} A^s \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t \right).$$

The allocation $\tilde{a}_{tD} = A^s/v$, $\tilde{a}_{tI} = 0$, and $\underline{a}_{tib(i)}\left(\varepsilon_i, \varepsilon_{b(i)}\right) = \mathbb{I}_{\{\varepsilon_{b(i)} < \varepsilon_i\}} 2a_{tI} + \mathbb{I}_{\{\varepsilon_{b(i)} = \varepsilon_i\}} a^o$, where $a^o \in [0, 2a_{tI}]$, together with the Dirac measure defined in the statement of the proposition, achieve $\bar{W}^*$ and therefore solve the Planner's problem. ■

Proof of Lemma 1. Notice that (8) can be written as

$$W_t^D(\boldsymbol{a}_t) = \boldsymbol{\phi}_t \boldsymbol{a}_t + W_t^D(\mathbf{0}) \tag{59}$$

with $W_t^D(\mathbf{0})$ given by (14). With (59), (9) is equivalent to

$$\hat{W}_t^D(\boldsymbol{a}_t) = \max_{\hat{a}_t^m, \hat{a}_t^s} [\phi_t^m \hat{a}_t^m + \phi_t^s \hat{a}_t^s + \xi(a_t^m + p_t a_t^s - \hat{a}_t^m - p_t \hat{a}_t^s) + \varsigma_m \hat{a}_t^m + \varsigma_s \hat{a}_t^s] + W_t^D(\mathbf{0})$$

where ξ is a Lagrange multiplier on the budget constraint $\hat{a}_t^m + p_t \hat{a}_t^s \leq a_t^m + p_t a_t^s$, and ς_m and ς_s are the multipliers on the nonnegativity constraints $\hat{a}_t^m \geq 0$ and $\hat{a}_t^s \geq 0$. The corresponding first-order necessary and sufficient conditions for $\hat{a}_t^m$ and $\hat{a}_t^s$ are

$$-\xi + \phi_t^m + \varsigma_m = 0 \tag{60}$$

$$-\xi p_t + \phi_t^s + \varsigma_s = 0 \tag{61}$$

$$\xi(a_t^m + p_t a_t^s - \hat{a}_t^m - p_t \hat{a}_t^s) = 0. \tag{62}$$

Clearly $\hat{a}_t^m = \hat{a}_t^s = 0$ is the solution if and only if $a_t^m = a_t^s = 0$, but more generally the solution could take one of three forms: (i) $\varsigma_s = 0 < \varsigma_m$, (ii) $\varsigma_s = \varsigma_m = 0$, or (iii) $\varsigma_m = 0 < \varsigma_s$. In case (i), (60)-(62) imply $\hat{a}_t^m = 0$, $\hat{a}_t^s = a_t^s + \frac{1}{p_t} a_t^m$, and $p_t \phi_t^m < \phi_t^s$. In case (ii), (60)-(62) imply $\hat{a}_t^m \in [0, a_t^m + p_t a_t^s]$, $\hat{a}_t^s = a_t^s + \frac{1}{p_t}(a_t^m - \hat{a}_t^m)$, and $\phi_t^s = p_t \phi_t^m$. In case (iii), (60)-(62) imply $\hat{a}_t^s = 0$, $\hat{a}_t^m = a_t^m + p_t a_t^s$, and $\phi_t^s < p_t \phi_t^m$. The expressions for $\hat{a}_{td}^m$ and $\hat{a}_{td}^s$ in Lemma 1 follow from these three cases. The value function (13) is obtained by substituting the optimal portfolio $(\hat{a}_{td}^m, \hat{a}_{td}^s)$ into (9). ■

Proof of Lemma 2. (i) Notice that (11) can be written as

$$W_t^I(\boldsymbol{a}_t) = \boldsymbol{\phi}_t \boldsymbol{a}_t + W_t^I(\mathbf{0}) \tag{63}$$

where

$$W_t^I(\mathbf{0}) = T_t + \max_{\tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2} \left[\beta \mathbb{E}_t \int V_{t+1}^I(\boldsymbol{a}_{t+1}, \varepsilon) \, dG(\varepsilon) - \boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} \right]$$
$$\text{s.t. } \boldsymbol{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s + (1-\pi) A^s).$$

With (13) and (63) the problem of the investor when he makes the ultimatum offer becomes

$$\max_{\overline{a}_{ti^*}^m, \overline{a}_{ti^*}^s, \overline{a}_{td}^m, \overline{a}_{td}^s} [\varepsilon y_t \overline{a}_{ti^*}^s + \phi_t^m \overline{a}_{ti^*}^m + \phi_t^s \overline{a}_{ti^*}^s]$$

$$\text{s.t. } \overline{a}_{ti^*}^m + \overline{a}_{td}^m + p_t(\overline{a}_{ti^*}^s + \overline{a}_{td}^s) \leq a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s)$$
$$\overline{a}_{td}^m + p_t \overline{a}_{td}^s \geq a_{td}^m + p_t a_{td}^s$$
$$\overline{a}_{ti^*}^m, \overline{a}_{ti^*}^s, \overline{a}_{td}^m, \overline{a}_{td}^s \in \mathbb{R}_+.$$

The corresponding Lagrangian is

$$\mathcal{L} = (\phi_t^m + \varsigma_i^m - \xi)\,\overline{a}_{ti^*}^m + (\varepsilon y_t + \phi_t^s + \varsigma_i^s - \xi p_t)\,\overline{a}_{ti^*}^s$$
$$+ (\rho + \varsigma_d^m - \xi)\,\overline{a}_{td}^m + (\rho p_t + \varsigma_d^s - \xi p_t)\,\overline{a}_{td}^s + K,$$

where $K \equiv \xi\,[a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s)] - \rho\,(a_{td}^m + p_t a_{td}^s)$, $\xi \in \mathbb{R}_+$ is the Lagrange multiplier associated with the budget constraint, $\rho \in \mathbb{R}_+$ is the multiplier on the dealer's individual rationality constraint, and $\varsigma_i^m, \varsigma_i^s, \varsigma_d^m, \varsigma_d^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\overline{a}_{ti^*}^m$, $\overline{a}_{ti^*}^s$, $\overline{a}_{td}^m$, $\overline{a}_{td}^s$, respectively. The first-order necessary and sufficient conditions are

$$\phi_t^m + \varsigma_i^m - \xi = 0 \tag{64}$$

$$\varepsilon y_t + \phi_t^s + \varsigma_i^s - \xi p_t = 0 \tag{65}$$

$$\rho + \varsigma_d^m - \xi = 0 \tag{66}$$

$$\rho p_t + \varsigma_d^s - \xi p_t = 0 \tag{67}$$

and the complementary slackness conditions

$$\xi\,\{a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s) - [\overline{a}_{ti^*}^m + \overline{a}_{td}^m + p_t(\overline{a}_{ti^*}^s + \overline{a}_{td}^s)]\} = 0 \tag{68}$$

$$\rho\,[\overline{a}_{td}^m + p_t\overline{a}_{td}^s - (a_{td}^m + p_t a_{td}^s)] = 0 \tag{69}$$

$$\varsigma_i^m\overline{a}_{ti^*}^m = 0 \tag{70}$$

$$\varsigma_i^s\overline{a}_{ti^*}^s = 0 \tag{71}$$

$$\varsigma_d^m\overline{a}_{td}^m = 0 \tag{72}$$

$$\varsigma_d^s\overline{a}_{td}^s = 0. \tag{73}$$

First, notice that $\xi > 0$ at an optimum. To see this, assume the contrary, i.e., $\xi = 0$. Then (65) implies $\varepsilon y_t + \phi_t^s = -\varsigma_i^s \leq 0$ which is a contradiction since $\varepsilon y_t + \phi_t^s > 0$. If $\rho > 0$, then (69) implies

$$\overline{a}_{td}^m + p_t\overline{a}_{td}^s = a_{td}^m + p_t a_{td}^s. \tag{74}$$

If instead $\rho = 0$, then (66) and (67) imply $\varsigma_d^m = \xi > 0$ and $\varsigma_d^s = \xi p_t > 0$, which (using (72) and (73)) in turn imply $\overline{a}_{td}^m = \overline{a}_{td}^s = 0$. This can only be a solution if $a_{td}^m + p_t a_{td}^s = 0$ (since $\overline{a}_{td}^m + p_t\overline{a}_{td}^s \geq a_{td}^m + p_t a_{td}^s$ must hold at an optimum) in which case (74) also holds. Thus, we conclude that (74) must always hold at an optimum (and with $\rho > 0$ unless $a_{td}^m + p_t a_{td}^s = 0$). Since $\xi > 0$, (68) and (74) imply

$$\overline{a}_{ti^*}^m + p_t\overline{a}_{ti^*}^s = a_{ti}^m + p_t a_{ti}^s. \tag{75}$$

From (74) it is immediate that if $a^m_{td}+p_t a^s_{td}=0$, then $\overline{a}^m_{td}=\overline{a}^s_{td}=0$. So suppose $a^m_{td}+p_t a^s_{td}>0$. In this case ς^m_d and ς^s_d cannot both be strictly positive. (To see this, assume the contrary, i.e., that $\varsigma^m_d>0$ and $\varsigma^s_d>0$. Then (72) and (73) imply $\overline{a}^m_{td}=\overline{a}^s_{td}=0$, and (74) implies $a^m_{td}+p_t a^s_{td}=0$, a contradiction.) Moreover, conditions (66) and (67) imply $\varsigma^s_d=\varsigma^m_d p_t$, so $\varsigma^s_d=\varsigma^m_d=0$ must hold at an optimum. Hence when making the ultimatum offer, the investor is indifferent between offering the dealer any nonnegative pair $(\overline{a}^m_{td},\overline{a}^s_{td})$ that satisfies (74).

From (75) it is immediate that $\overline{a}^m_{ti^*}=\overline{a}^s_{ti^*}=0$ if $a^m_{ti}+p_t a^s_{ti}=0$. So suppose $a^m_{ti}+p_t a^s_{ti}>0$. In this case ς^m_i and ς^s_i cannot both be strictly positive (if they were, then (70) and (71) would imply $\overline{a}^m_{ti^*}=\overline{a}^s_{ti^*}=0$, and in turn (75) would imply $a^m_{ti}+p_t a^s_{ti}=0$, a contradiction). There are three possible cases: (a) $\varsigma^s_i=0<\varsigma^m_i$, (b) $\varsigma^s_i=\varsigma^m_i=0$, or (c) $\varsigma^m_i=0<\varsigma^s_i$. In every case, (64) and (65) imply

$$\varepsilon y_t+\phi^s_t+\varsigma^s_i=p_t\phi^m_t+p_t\varsigma^m_i. \tag{76}$$

In case (a), (70) implies $\overline{a}^m_{ti^*}=0$, (75) implies $\overline{a}^s_{ti^*}=a^m_{ti}/p_t+a^s_{ti}$, and (76) implies that ε must satisfy $\varepsilon>\varepsilon^*_t$, where ε^*_t is as defined in (15). In case (b), (76) implies that ε must satisfy $\varepsilon=\varepsilon^*_t$ and the investor is indifferent between making any offer that leaves him with a nonnegative post-trade portfolio $(\overline{a}^m_{ti^*},\overline{a}^s_{ti^*})$ that satisfies (75). In case (c), (71) implies $\overline{a}^s_{ti^*}=0$, (75) implies $\overline{a}^m_{ti^*}=a^m_{ti}+p_t a^s_{ti}$, and (76) implies that ε must satisfy $\varepsilon<\varepsilon^*_t$. The first, second, and third lines on the right side of the expressions for $\overline{a}^m_{ti^*}$, $\overline{a}^s_{ti^*}$, $\overline{a}^m_{td}$, and $\overline{a}^s_{td}$ in part (i) of the statement of the lemma correspond cases (a), (b), and (c), respectively.

(ii) With (13) and (63) the problem of the dealer when it is his turn to make the ultimatum offer is equivalent to

$$\max_{\overline{a}^m_{ti},\overline{a}^s_{ti},\overline{a}^m_{td^*},\overline{a}^s_{td^*}} \bar{\phi}_t\left[\overline{a}^m_{td^*}+p_t\overline{a}^s_{td^*}\right]$$

$$\text{s.t. } \overline{a}^m_{ti}+\overline{a}^m_{td^*}+p_t(\overline{a}^s_{ti}+\overline{a}^s_{td^*})\le a^m_{ti}+a^m_{td}+p_t(a^s_{ti}+a^s_{td}) \tag{77}$$

$$\phi^m_t\overline{a}^m_{ti}+(\varepsilon y_t+\phi^s_t)\,\overline{a}^s_{ti}\ge\phi^m_t a^m_{ti}+(\varepsilon y_t+\phi^s_t)\,a^s_{ti} \tag{78}$$

$$\overline{a}^m_{ti},\overline{a}^s_{ti},\overline{a}^m_{td^*},\overline{a}^s_{td^*}\in\mathbb{R}_+.$$

The corresponding Lagrangian is

$$\begin{aligned}\mathcal{L}'=&(\bar{\phi}_t+\varsigma^m_d-\xi)\overline{a}^m_{td^*}+(\bar{\phi}_t p_t+\varsigma^s_d-\xi p_t)\overline{a}^s_{td^*}\\&+(\rho\phi^m_t+\varsigma^m_i-\xi)\,\overline{a}^m_{ti}+\left[\rho\left(\varepsilon y_t+\phi^s_t\right)+\varsigma^s_i-\xi p_t\right]\overline{a}^s_{ti}+K',\end{aligned}$$

where $K'\equiv\xi\left[a^m_{ti}+a^m_{td}+p_t\left(a^s_{ti}+a^s_{td}\right)\right]-\rho\left[\phi^m_t a^m_{ti}+\left(\varepsilon y_t+\phi^s_t\right)a^s_{ti}\right]$, $\xi\in\mathbb{R}_+$ is the Lagrange multiplier associated with the budget constraint, $\rho\in\mathbb{R}_+$ is the multiplier on the investor's individual rationality constraint, and $\varsigma_i^m, \varsigma_i^s, \varsigma_d^m, \varsigma_d^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\overline{a}_{ti}^m$, $\overline{a}_{ti}^s$, $\overline{a}_{td^*}^m$, $\overline{a}_{td^*}^s$, respectively. The first-order necessary and sufficient conditions are

$$\bar{\phi}_t + \varsigma_d^m - \xi = 0 \tag{79}$$

$$\bar{\phi}_t p_t + \varsigma_d^s - \xi p_t = 0 \tag{80}$$

$$\rho \phi_t^m + \varsigma_i^m - \xi = 0 \tag{81}$$

$$\rho \left(\varepsilon y_t + \phi_t^s \right) + \varsigma_i^s - \xi p_t = 0 \tag{82}$$

and the complementary slackness conditions

$$\xi \left\{ a_{ti}^m + a_{td}^m + p_t (a_{ti}^s + a_{td}^s) - \left[\overline{a}_{ti}^m + \overline{a}_{td^*}^m + p_t (\overline{a}_{ti}^s + \overline{a}_{td^*}^s) \right] \right\} = 0 \tag{83}$$

$$\rho \left\{ \phi_t^m \overline{a}_{ti}^m + \left(\varepsilon y + \phi_t^s \right) \overline{a}_{ti}^s - \left[\phi_t^m a_{ti}^m + \left(\varepsilon y_t + \phi_t^s \right) a_{ti}^s \right] \right\} = 0 \tag{84}$$

$$\varsigma_i^m \overline{a}_{ti}^m = 0 \tag{85}$$

$$\varsigma_i^s \overline{a}_{ti}^s = 0 \tag{86}$$

$$\varsigma_d^m \overline{a}_{td^*}^m = 0 \tag{87}$$

$$\varsigma_d^s \overline{a}_{td^*}^s = 0. \tag{88}$$

First, notice that $\xi > 0$ at an optimum. To see this, note that if $\xi = 0$ then (79) implies $\bar{\phi}_t + \varsigma_d^m = 0$ which is a contradiction since the left side is strictly positive ($\bar{\phi}_t > 0$ and $\varsigma_d^m \geq 0$ in a monetary equilibrium). Hence, at an optimum,

$$\overline{a}_{ti}^m + \overline{a}_{td^*}^m + p_t (\overline{a}_{ti}^s + \overline{a}_{td^*}^s) = a_{ti}^m + a_{td}^m + p_t (a_{ti}^s + a_{td}^s). \tag{89}$$

Second, observe that conditions (79) and (80), imply $p_t \varsigma_d^m = \varsigma_d^s$, so ς_d^m and ς_d^s have the same sign, i.e., either both are positive or both are zero.

If $\rho = 0$, then (81) and (82) imply $\varsigma_i^m = \xi > 0$ and $\varsigma_i^s = \xi p_t > 0$, which (using (85) and (86)) in turn imply $\overline{a}_{ti}^m = \overline{a}_{ti}^s = 0$. From the buyer's individual rationality constraint (78) it follows that this can be a solution only if $\phi_t^m a_{ti}^m + \left(\varepsilon y_t + \phi_t^s \right) a_{ti}^s = 0$, or equivalently only if $a_{ti}^m = a_{ti}^s = 0$. To obtain $(\overline{a}_{td^*}^m, \overline{a}_{td^*}^s)$, consider two cases: (a) $\varsigma_d^m = \varsigma_d^s = 0$, in which case $(\overline{a}_{td^*}^m, \overline{a}_{td^*}^s)$ need only satisfy $\overline{a}_{td^*}^m + p_t \overline{a}_{td^*}^s = a_{td}^m + p_t a_{td}^s$, or (b) $\varsigma_d^m > 0$ and $\varsigma_d^s > 0$, in which case $\overline{a}_{td^*}^m = \overline{a}_{td^*}^s = 0$, which according to (77), is only possible if $a_{td}^m = a_{td}^s = 0$. It is easy to see that the solution for case (a) can be obtained from the expressions for $\overline{a}_{ti}^m$, $\overline{a}_{ti}^s$, $\overline{a}_{td^*}^m$, and $\overline{a}_{td^*}^s$ in part (ii) of the statement of the lemma simply by setting $a^m_{ti} = a^s_{ti} = 0$, and the solution for case (b) can be obtained similarly, by setting $a^m_{ti} = a^s_{ti} = a^m_{td} = a^s_{td} = 0$.

If $\rho > 0$, then (84) implies

$$\phi^m_t \overline{a}^m_{ti} + (\varepsilon y_t + \phi^s_t)\, \overline{a}^s_{ti} = \phi^m_t a^m_{ti} + (\varepsilon y + \phi^s_t)\, a^s_{ti}. \tag{90}$$

There are eight possible configurations of to be considered: [Configuration 1] $\varsigma^s_i = \varsigma^m_d = \varsigma^s_d = 0 < \varsigma^m_i$. In this case (85) implies $\overline{a}^m_{ti} = 0$. Conditions (79)-(82) imply $\varsigma^m_i = (\varepsilon - \varepsilon^*_t)\, \bar{\phi}_t y_t / (\varepsilon y_t + \phi^s_t)$, and therefore $\varepsilon^*_t < \varepsilon$. Then from (89) and (90) it follows that

$$\overline{a}^s_{ti} = a^s_{ti} + \left(\frac{\varepsilon^*_t y_t + \phi^s_t}{\varepsilon y_t + \phi^s_t} \right) \frac{1}{p_t} a^m_{ti}$$

and $(\overline{a}^m_{td^*}, \overline{a}^s_{td^*})$ is any nonnegative pair that satisfies

$$\overline{a}^m_{td^*} + p_t \overline{a}^s_{td^*} = a^m_{td} + p_t a^s_{td} + \frac{(\varepsilon - \varepsilon^*_t)\, y_t}{\varepsilon y_t + \phi^s_t} a^m_{ti}.$$

[Configuration 2] $\varsigma^m_i = \varsigma^s_i = \varsigma^m_d = \varsigma^s_d = 0$. In this case conditions (79)-(82) imply $\varepsilon = \varepsilon^*_t$, and (89) and (90) yield

$$\overline{a}^m_{ti} + p_t \overline{a}^s_{ti} = a^m_{ti} + p_t a^s_{ti} \tag{91}$$

$$\overline{a}^m_{td^*} + p_t \overline{a}^s_{td^*} = a^m_{td} + p_t a^s_{td}. \tag{92}$$

Hence the dealer is indifferent between making any offer $(\overline{a}^m_{ti}, \overline{a}^s_{ti}, \overline{a}^m_{td^*}, \overline{a}^s_{td^*})$ such that $(\overline{a}^m_{ti}, \overline{a}^s_{ti}) \in \mathbb{R}_+$ satisfies (91), and $(\overline{a}^m_{td^*}, \overline{a}^s_{td^*}) \in \mathbb{R}_+$ satisfies (92). [Configuration 3] $\varsigma^m_i = \varsigma^m_d = \varsigma^s_d = 0 < \varsigma^s_i$. In this case condition (86) implies $\overline{a}^s_{ti} = 0$. Conditions (81) and (82) imply $\varsigma^s_i = (\varepsilon^*_t - \varepsilon)\, y_t \rho$, and therefore $\varepsilon < \varepsilon^*_t$. Then from (89) and (90) it follows that

$$\overline{a}^m_{ti} = a^m_{ti} + \frac{\varepsilon y_t + \phi^s_t}{\varepsilon^*_t y_t + \phi^s_t} p_t a^s_{ti}$$

and $(\overline{a}^m_{td^*}, \overline{a}^s_{td^*})$ is any nonnegative pair that satisfies

$$\overline{a}^m_{td^*} + p_t \overline{a}^s_{td^*} = a^m_{td} + p_t a^s_{td} + \frac{(\varepsilon^*_t - \varepsilon)\, y_t}{\varepsilon^*_t y_t + \phi^s_t} p_t a^s_{ti}.$$

[Configuration 4] $\varsigma^m_d = \varsigma^s_d = 0$, $0 < \varsigma^m_i$ and $0 < \varsigma^s_i$. In this case conditions (85) and (86) imply $\overline{a}^m_{ti} = \overline{a}^s_{ti} = 0$, which according to (90), is only possible if $a^m_{ti} = a^s_{ti} = 0$. Then $(\overline{a}^m_{td^*}, \overline{a}^s_{td^*})$ is any nonnegative pair that satisfies (92). [Configuration 5] $\varsigma^s_i = 0 < \varsigma^m_i$, $0 < \varsigma^m_d$ and $0 < \varsigma^s_d$. In this case conditions (85), (87) and (88) imply $\overline{a}^m_{ti} = \overline{a}^m_{td^*} = \overline{a}^s_{td^*} = 0$. Conditions (81) and (82) imply $\varepsilon^*_t < \varepsilon$. Then from (89) and (90) it follows that the following condition must hold:

$$a^s_{td} + \frac{1}{p_t} a^m_{td} = -\left[\frac{(\varepsilon - \varepsilon^*_t)\, y_t}{(\varepsilon - \varepsilon^*_t)\, y_t + p_t \phi^m_t}\right] \frac{1}{p_t} a^m_{ti}.$$

The term on the left side of the equality is nonnegative and the term on the right side of the equality is nonpositive (since $\varepsilon^*_t < \varepsilon$), so this condition can hold only if $a^m_{ti} = a^m_{td} = a^s_{td} = 0$. Therefore (89) implies $\overline{a}^s_{ti} = a^s_{ti}$. [Configuration 6] $\varsigma^m_i = \varsigma^s_i = 0$, $0 < \varsigma^m_d$ and $0 < \varsigma^s_d$. In this case conditions (87) and (88) imply $\overline{a}^m_{td^*} = \overline{a}^s_{td^*} = 0$. Conditions (81) and (82) imply $\varepsilon = \varepsilon^*_t$, and in turn conditions (89) and (90) imply $a^m_{td} + p_t a^s_{td} = 0$, or equivalently, $a^m_{td} = a^s_{td} = 0$ must hold, and $(\overline{a}^m_{ti}, \overline{a}^s_{ti})$ is any nonnegative pair that satisfies (91). [Configuration 7] $\varsigma^m_i = 0 < \varsigma^s_i$, $0 < \varsigma^s_d$ and $0 < \varsigma^s_d$. In this case conditions (86)-(88) imply $\overline{a}^s_{ti} = \overline{a}^m_{td^*} = \overline{a}^s_{td^*} = 0$. Conditions (81) and (82) imply $\varepsilon < \varepsilon^*_t$. Then from (89) and (90) it follows that the following condition must hold:

$$\phi^m_t \left(a^m_{td} + p_t a^s_{td}\right) = -\left(\varepsilon^*_t - \varepsilon\right) y_t a^s_{ti}.$$

The term on the left side of the equality is nonnegative and the term on the right side of the equality is nonpositive (since $\varepsilon < \varepsilon^*_t$), so this condition can hold only if $\phi^m_t \left(a^m_{td} + p_t a^s_{td}\right) = a^s_{ti} = 0$. Therefore (90) implies $\overline{a}^m_{ti} = a^m_{ti}$. [Configuration 8] $0 < \varsigma^m_i$, $0 < \varsigma^s_i$, $0 < \varsigma^s_d$ and $0 < \varsigma^s_d$. In this case conditions (85)-(88) imply $\overline{a}^m_{ti} = \overline{a}^s_{ti} = \overline{a}^m_{td^*} = \overline{a}^s_{td^*} = 0$, which according to (89) is only possible, and the only possible solution if $a^m_{ti} = a^s_{ti} = a^m_{td} = a^s_{td} = 0$. To conclude, notice that the solutions for Configurations 1, 2, and 3, correspond to the first, second, and third lines of the expressions for $\overline{a}^m_{ti}$, $\overline{a}^s_{ti}$, $\overline{a}^m_{td^*}$, and $\overline{a}^s_{td^*}$ in part (*ii*) of the statement of the lemma. Similarly, the solution for Configuration 5 corresponds to the first line of the expressions for $\overline{a}^m_{ti}$, $\overline{a}^s_{ti}$, $\overline{a}^m_{td^*}$, and $\overline{a}^s_{td^*}$ in part (*ii*) of the statement of the lemma, with $a^m_{ti} = a^m_{td} = a^s_{td} = 0$. The solution for Configuration 6 corresponds to the second line of the expressions for $\overline{a}^m_{ti}$, $\overline{a}^s_{ti}$, $\overline{a}^m_{td^*}$, and $\overline{a}^s_{td^*}$ in part (*ii*) of the statement of the lemma, with $a^m_{td} = a^s_{td} = 0$. The solution for Configuration 7 corresponds to the third line of the expressions for $\overline{a}^m_{ti}$, $\overline{a}^s_{ti}$, $\overline{a}^m_{td^*}$, and $\overline{a}^s_{td^*}$ in part (*ii*) of the statement of the lemma, with $\phi^m_t \left(a^m_{td} + p_t a^s_{td}\right) = a^s_{ti} = 0$. Finally, it is easy to see that the solution for Configuration 4 can be obtained from the expressions for $\overline{a}^m_{ti}$, $\overline{a}^s_{ti}$, $\overline{a}^m_{td^*}$, and $\overline{a}^s_{td^*}$ in part (*ii*) of the statement of the lemma simply by setting $a^m_{ti} = a^s_{ti} = 0$, and the solution for case Configuration 8 can be obtained similarly, by setting $a^m_{ti} = a^s_{ti} = a^m_{td} = a^s_{td} = 0$. ■

Proof of Lemma 3. With (63) investor i's problem when choosing his take-it-or-leave it offer to investor j reduces to

$$\max_{\underline{a}_{ti*}^{m},\underline{a}_{ti*}^{s},\underline{a}_{tj}^{m},\underline{a}_{tj}^{s}} \left[\left(\varepsilon_i y_t + \phi_t^s\right)\underline{a}_{ti*}^{s} + \phi_t^m \underline{a}_{ti*}^{m}\right]$$

$$\text{s.t. } \underline{a}_{ti*}^{m} + \underline{a}_{tj}^{m} \leq a_{ti}^{m} + a_{tj}^{m}$$

$$\underline{a}_{ti*}^{s} + \underline{a}_{tj}^{s} \leq a_{ti}^{s} + a_{tj}^{s}$$

$$\varepsilon_j y_t \underline{a}_{tj}^{s} + \phi_t^m \underline{a}_{tj}^{m} + \phi_t^s \underline{a}_{tj}^{s} \geq \varepsilon_j y_t a_{tj}^{s} + \phi_t^m a_{tj}^{m} + \phi_t^s a_{tj}^{s}$$

$$\underline{a}_{ti*}^{m}, \underline{a}_{ti*}^{s}, \underline{a}_{tj}^{m}, \underline{a}_{tj}^{s} \in \mathbb{R}_{+}.$$

If $\phi_t^m = 0$, then $\underline{a}_{ti*}^{s} = a_{ti}^{s}$ and $\underline{a}_{tj}^{s} = a_{tj}^{s}$ (the bargaining outcome is no trade between investors i and j) so suppose $\phi_t^m > 0$ for the rest of the proof. The Lagrangian corresponding to investor i's problem is

$$\mathcal{L} = \left(\phi_t^m + \varsigma_i^m - \xi^m\right)\underline{a}_{ti*}^{m} + \left(\varepsilon_i y_t + \phi_t^s + \varsigma_i^s - \xi^s\right)\underline{a}_{ti*}^{s}$$
$$+ \left(\rho\phi_t^m + \varsigma_j^m - \xi^m\right)\underline{a}_{tj}^{m} + \left[\rho\left(\varepsilon_j y_t + \phi_t^s\right) + \varsigma_j^s - \xi^s\right]\underline{a}_{tj}^{s} + K'',$$

where $K'' \equiv \xi^m(a_{ti}^m + a_{tj}^m) + \xi^s(a_{ti}^s + a_{tj}^s) - \rho(\varepsilon_j y_t a_{tj}^s + \phi_t^m a_{tj}^m + \phi_t^s a_{tj}^s)$, $\xi^m \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on money holdings, $\xi^s \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on equity holdings, $\rho \in \mathbb{R}_+$ is the multiplier on investor j's individual rationality constraint, and $\varsigma_i^m, \varsigma_i^s, \varsigma_j^m, \varsigma_j^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\overline{a}_{ti*}^{m}$, $\overline{a}_{ti*}^{s}$, $\overline{a}_{tj}^{m}$, $\overline{a}_{tj}^{s}$, respectively. The first-order necessary and sufficient conditions are

$$\phi_t^m + \varsigma_i^m - \xi^m = 0 \tag{93}$$

$$\varepsilon_i y_t + \phi_t^s + \varsigma_i^s - \xi^s = 0 \tag{94}$$

$$\rho\phi_t^m + \varsigma_j^m - \xi^m = 0 \tag{95}$$

$$\rho\left(\varepsilon_j y_t + \phi_t^s\right) + \varsigma_j^s - \xi^s = 0 \tag{96}$$

and the complementary slackness conditions

$$\xi^m(a^m_{ti} + a^m_{tj} - \underline{a}^m_{ti^*} - \underline{a}^m_{tj}) = 0 \tag{97}$$

$$\xi^s(a^s_{ti} + a^s_{tj} - \underline{a}^s_{ti^*} - \underline{a}^s_{tj}) = 0 \tag{98}$$

$$\rho(\varepsilon_j y_t \underline{a}^s_{tj} + \phi^m_t \underline{a}^m_{tj} + \phi^s_t \underline{a}^s_{tj} - \varepsilon_j y_t a^s_{tj} - \phi^m_t a^m_{tj} - \phi^s_t a^s_{tj}) = 0 \tag{99}$$

$$\varsigma^m_i \underline{a}^m_{ti^*} = 0 \tag{100}$$

$$\varsigma^s_i \underline{a}^s_{ti^*} = 0 \tag{101}$$

$$\varsigma^m_j \underline{a}^m_{tj} = 0 \tag{102}$$

$$\varsigma^s_j \underline{a}^s_{tj} = 0. \tag{103}$$

If $\xi^m = 0$, (93) implies $0 < \phi^m_t = -\varsigma^m_i \leq 0$, a contradiction. If $\xi^s = 0$, (94) implies $0 < \varepsilon_i y_t + \phi^s_t = -\varsigma^s_i \leq 0$, another contradiction. Hence $\xi^m > 0$ and $\xi^s > 0$, so (97) and (98) imply

$$\underline{a}^m_{ti^*} + \underline{a}^m_{tj} = a^m_{ti} + a^m_{tj} \tag{104}$$

$$\underline{a}^s_{ti^*} + \underline{a}^s_{tj} = a^s_{ti} + a^s_{tj}. \tag{105}$$

If $\rho = 0$, (95) and (96) imply $\varsigma^m_j = \xi^m > 0$ and $\varsigma^s_j = \xi^s > 0$, and (102) and (103) imply $\underline{a}^m_{tj} = \underline{a}^s_{tj} = 0$. From investor's j individual rationality constraint, this can only be a solution if $a^m_{tj} = a^s_{tj} = 0$, and if this is the case (97) and (98) imply $(\underline{a}^m_{ti^*}, \underline{a}^s_{ti^*}) = (a^m_{ti}, a^s_{ti})$. Hereafter suppose $\rho > 0$ which using (99) implies

$$\phi^m_t \underline{a}^m_{tj} + (\varepsilon_j y_t + \phi^s_t)\underline{a}^s_{tj} = \phi^m_t a^m_{tj} + (\varepsilon_j y_t + \phi^s_t) a^s_{tj}. \tag{106}$$

If $\varsigma^m_i > 0$ and $\varsigma^m_j > 0$, (100) and (102) imply $\underline{a}^m_{ti^*} = \underline{a}^m_{tj} = 0$ which by (104), is only possible if $a^m_{ti} = a^m_{tj} = 0$. But then (106) implies $\underline{a}^s_{tj} = a^s_{tj}$, and (105) implies $\underline{a}^s_{ti^*} = a^s_{ti}$. Similarly, if $\varsigma^s_i > 0$ and $\varsigma^s_j > 0$, (101) and (103) imply $\underline{a}^s_{ti^*} = \underline{a}^s_{tj} = 0$ which by (105), is only possible if $a^s_{ti} = a^s_{tj} = 0$. But then (106) implies $\underline{a}^m_{tj} = a^m_{tj}$, and (104) implies $\underline{a}^m_{ti^*} = a^m_{ti}$. If $\varsigma^m_i > 0$ and $\varsigma^s_i > 0$, then (100) and (101) imply $\underline{a}^m_{ti^*} = \underline{a}^s_{ti^*} = 0$, and according to (104), (105) and (106), this is only possible if $a^m_{ti} = a^s_{ti} = 0$. Conditions (104) and (105) in turn imply $(\underline{a}^m_{tj}, \underline{a}^s_{tj}) = (a^m_{tj}, a^s_{tj})$. Similarly, if $\varsigma^m_j > 0$ and $\varsigma^s_j > 0$, then (102) and (103) imply $\underline{a}^m_{tj} = \underline{a}^s_{tj} = 0$, and according to (106) this is only possible if $a^m_{tj} = a^s_{tj} = 0$. Conditions (104) and (105) in turn imply $(\underline{a}^m_{ti^*}, \underline{a}^s_{ti^*}) = (a^m_{ti}, a^s_{ti})$. So far we have simply verified that there is no trade between investors i and j, i.e., $(\underline{a}^m_{ti^*}, \underline{a}^s_{ti^*}) = (a^m_{ti}, a^s_{ti})$ and $(\underline{a}^m_{tj}, \underline{a}^s_{tj}) = (a^m_{tj}, a^s_{tj})$, if $a^m_{ti} = a^m_{tj} = 0$, or $a^s_{ti} = a^s_{tj} = 0$, or

$a_{ti}^m = a_{ti}^s = 0$, or $a_{tj}^m = a_{tj}^s = 0$. Thus there are seven binding patterns for $(\varsigma_i^m, \varsigma_i^s, \varsigma_j^m, \varsigma_j^s)$ that remain to be considered.

(i) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^m = \varsigma_j^s = 0$. Conditions (93)-(96) imply that this case is only possible if $\varepsilon_i = \varepsilon_j$, and conditions (104), (105) and (106), imply that the solution consists of any pair of post trade portfolios $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$ and $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$ that satisfy

$$\begin{aligned}
\underline{a}_{tj}^m &= a_{tj}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m}\left(a_{ti}^s - \underline{a}_{ti^*}^s\right) \\
\underline{a}_{ti^*}^m &= a_{ti}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m}\left(a_{ti}^s - \underline{a}_{ti^*}^s\right) \\
\underline{a}_{tj}^s &= a_{ti}^s + a_{tj}^s - \underline{a}_{ti^*}^s \\
\underline{a}_{ti^*}^s &\in \left[a_{ti}^s - \min\left(\frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m, a_{ti}^s\right), a_{ti}^s + \min\left(\frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m, a_{tj}^s\right)\right].
\end{aligned}$$

(ii) $\varsigma_i^s = \varsigma_j^m = \varsigma_j^s = 0 < \varsigma_i^m$. Condition (100) implies $\underline{a}_{ti^*}^m = 0$, and from (104) we obtain $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$. Then condition (106) yields

$$\underline{a}_{tj}^s = a_{tj}^s - \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m$$

and condition (105) implies

$$\underline{a}_{ti^*}^s = a_{ti}^s + \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{ti}^m.$$

Notice that $\varsigma_j^s = 0$ requires $\underline{a}_{tj}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{ti}^m \leq \left(\varepsilon_j y_t + \phi_t^s\right) a_{tj}^s.$$

Conditions (93)-(96) imply $\varsigma_i^m = \left(\varepsilon_i - \varepsilon_j\right) y_t \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s}$, so $\varsigma_i^m > 0$ requires $\varepsilon_j < \varepsilon_i$.

(iii) $\varsigma_i^m = \varsigma_j^m = \varsigma_j^s = 0 < \varsigma_i^s$. Condition (101) implies $\underline{a}_{ti^*}^s = 0$, and from (105) we obtain $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Then condition (106) yields

$$\underline{a}_{tj}^m = a_{tj}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{ti}^s$$

and condition (104) implies

$$\underline{a}_{ti^*}^m = a_{ti}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{ti}^s.$$

Notice that $\varsigma_j^m = 0$ requires $\underline{a}_{tj}^m \geq 0$ which is equivalent to

$$\left(\varepsilon_j y_t + \phi_t^s\right) a_{ti}^s \leq \phi_t^m a_{tj}^m.$$

Conditions (93)-(96) imply $\varsigma_i^s = (\varepsilon_j - \varepsilon_i)\, y_t$, so $\varsigma_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$.

(*iv*) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^s = 0 < \varsigma_j^m$. Condition (102) implies $\underline{a}_{tj}^m = 0$, and from (104) we obtain $\underline{a}_{ti^*}^m = a_{ti}^m + a_{tj}^m$. Then (105) and (106) imply

$$\underline{a}_{tj}^s = a_{tj}^s + \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m$$
$$\underline{a}_{ti^*}^s = a_{ti}^s - \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s} a_{tj}^m.$$

Notice that $\varsigma_i^s = 0$ requires $\underline{a}_{ti^*}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{tj}^m \leq (\varepsilon_j y_t + \phi_t^s)\, a_{ti}^s.$$

Conditions (93)-(96) imply $\varsigma_j^m = (\varepsilon_j - \varepsilon_i)\, y_t \frac{\phi_t^m}{\varepsilon_j y_t + \phi_t^s}$, so $\varsigma_j^m > 0$ requires $\varepsilon_i < \varepsilon_j$.

(*v*) $\varsigma_i^m = \varsigma_i^s = \varsigma_j^m = 0 < \varsigma_j^s$. Condition (103) implies $\underline{a}_{tj}^s = 0$, and from (105) we obtain $\underline{a}_{ti^*}^s = a_{ti}^s + a_{tj}^s$. Then (104) and (106) imply

$$\underline{a}_{tj}^m = a_{tj}^m + \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{tj}^s$$
$$\underline{a}_{ti^*}^m = a_{ti}^m - \frac{\varepsilon_j y_t + \phi_t^s}{\phi_t^m} a_{tj}^s.$$

Notice that $\varsigma_i^m = 0$ requires $\underline{a}_{ti^*}^m \geq 0$ which is equivalent to

$$(\varepsilon_j y_t + \phi_t^s)\, a_{tj}^s \leq \phi_t^m a_{ti}^m.$$

Conditions (93)-(96) imply $\varsigma_j^s = (\varepsilon_i - \varepsilon_j)\, y_t$, so $\varsigma_j^s > 0$ requires $\varepsilon_j < \varepsilon_i$.

(*vi*) $\varsigma_i^m, \varsigma_j^s \in \mathbb{R}_{++}$ and $\varsigma_i^s = \varsigma_j^m = 0$. In this case, conditions (100) and (103) give $\underline{a}_{ti^*}^m = \underline{a}_{tj}^s = 0$, and (104) and (105) imply $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{ti^*}^s = a_{ti}^s + a_{tj}^s$. Condition (106) implies the following restriction must be satisfied

$$\phi_t^m a_{ti}^m = (\varepsilon_j y_t + \phi_t^s) a_{tj}^s.$$

Conditions (93)-(96) imply $\varsigma_i^m = (\rho - 1)\, \phi_t^m$ and $\varsigma_j^s = (\varepsilon_i - \varepsilon_j)\, y_t - (\rho - 1)\, (\varepsilon_j y_t + \phi_t^s)$, so $\varsigma_i^m > 0$ requires $\rho > 1$, and ς_j^s requires $\varepsilon_j < \varepsilon_i$.

(*vii*) $\varsigma_i^m = \varsigma_j^s = 0$ and $\varsigma_i^s, \varsigma_j^m \in \mathbb{R}_{++}$. In this case, conditions (101) and (102) give $\underline{a}_{ti^*}^s = \underline{a}_{tj}^m = 0$, and (104) and (105) imply $\underline{a}_{ti^*}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Condition (106) implies the following restriction must be satisfied

$$\phi_t^m a_{tj}^m = (\varepsilon_j y_t + \phi_t^s) a_{ti}^s.$$

Conditions (93)-(96) imply $\varsigma_j^m = (1-\rho)\,\phi_t^m$ and $\varsigma_i^s = (\varepsilon_j - \varepsilon_i)\,y_t - (1-\rho)\,(\varepsilon_j y_t + \phi_t^s)$, so $\varsigma_j^m > 0$ requires $\rho \in (0,1)$, and $\varsigma_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$. ■

Proof of Lemma 4. (i) With Lemma 1, (10) becomes

$$\begin{aligned} V_t^D\left(a_{td}^m, a_{td}^s\right) = {} & \kappa\theta \int \bar{\phi}_t \left[\overline{a}_{td}^m + p_t \overline{a}_{td}^s - \left(a_{td}^m + p_t a_{td}^s\right)\right] dH_t\left(\boldsymbol{a}_{ti}, \varepsilon\right) \\ & + \kappa\left(1-\theta\right) \int \bar{\phi}_t \left[\overline{a}_{td^*}^m + p_t \overline{a}_{td^*}^s - \left(a_{td}^m + p_t a_{td}^s\right)\right] dH_t\left(\boldsymbol{a}_{ti}, \varepsilon\right) \\ & + \bar{\phi}_t\left(a_{td}^m + p_t a_{td}^s\right) + W_t^D\left(\mathbf{0}\right) \end{aligned}$$

where we have used the more compact notation introduced in Lemma 2, i.e., $\overline{a}_{ti^*}^k \equiv \overline{a}_{i^*}^k(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, $\overline{a}_{td}^k \equiv \overline{a}_d^k(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, $\overline{a}_{ti}^k \equiv \overline{a}_i^k(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, and $\overline{a}_{td^*}^k \equiv \overline{a}_{d^*}^k(\boldsymbol{a}_{ti}, \boldsymbol{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, for $k = m, s$. Use Corollary 1 to arrive at

$$\begin{aligned} V_t^D\left(a_{td}^m, a_{td}^s\right) = {} & \kappa\left(1-\theta\right) \int \bar{\phi}_t \left[\mathbb{I}_{\{\varepsilon<\varepsilon_t^*\}} \frac{\left(\varepsilon_t^* - \varepsilon\right) y_t}{\varepsilon_t^* y_t + \phi_t^s} p_t a_{ti}^s + \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon\}} \frac{\left(\varepsilon - \varepsilon_t^*\right) y_t}{\varepsilon y_t + \phi_t^s} a_{ti}^m\right] dH_t\left(\boldsymbol{a}_{ti}, \varepsilon\right) \\ & + \bar{\phi}_t\left(a_{td}^m + p_t a_{td}^s\right) + W_t^D\left(\mathbf{0}\right) \end{aligned}$$

where $\mathbb{I}_{\{\varepsilon<\varepsilon_t^*\}}$ is an indicator function that takes the value 1 if $\varepsilon < \varepsilon_t^*$, and 0 otherwise. To obtain (17), use the fact that $dH_t\left(\boldsymbol{a}_{ti}, \varepsilon\right) = dF_t^I\left(\boldsymbol{a}_{ti}\right) dG\left(\varepsilon\right)$.

(ii) With (63) and the notation introduced in Lemma 2 and Lemma 3, (12) becomes

$$\begin{aligned} V_t^I\left(a_{ti}^m, a_{ti}^s, \varepsilon_i\right) = {} & \delta\theta \int \left[\phi_t^m\left(\overline{a}_{ti^*}^m - a_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right)\left(\overline{a}_{ti^*}^s - a_{ti}^s\right)\right] dF_t^D\left(\boldsymbol{a}_{td}\right) \\ & + \delta\left(1-\theta\right) \int \left[\phi_t^m\left(\overline{a}_{ti}^m - \overline{a}_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right)\left(\overline{a}_{ti}^s - a_{ti}^s\right)\right] dF_t^D\left(\boldsymbol{a}_{td}\right) \\ & + \alpha \int \tilde{\eta}\left(\varepsilon_i, \varepsilon_j\right)\left[\phi_t^m\left(\underline{a}_{ti^*}^m - a_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right)\left(\underline{a}_{ti^*}^s - a_{ti}^s\right)\right] dH_t\left(\boldsymbol{a}_{tj}, \varepsilon_j\right) \\ & + \alpha \int \left[1 - \tilde{\eta}\left(\varepsilon_i, \varepsilon_j\right)\right]\left[\phi_t^m\left(\underline{a}_{ti}^m - a_{ti}^m\right) + \left(\varepsilon_i y_t + \phi_t^s\right)\left(\underline{a}_{ti}^s - a_{ti}^s\right)\right] dH_t\left(\boldsymbol{a}_{tj}, \varepsilon_j\right) \\ & + \phi_t^m a_{ti}^m + \left(\varepsilon_i y_t + \phi_t^s\right) a_{ti}^s + W_t^I\left(\mathbf{0}\right). \end{aligned}$$

Use $\tilde{\eta}\left(\varepsilon_i, \varepsilon_j\right) \equiv \eta \mathbb{I}_{\{\varepsilon_j<\varepsilon_i\}} + \left(1-\eta\right) \mathbb{I}_{\{\varepsilon_i<\varepsilon_j\}} + \left(1/2\right) \mathbb{I}_{\{\varepsilon_i=\varepsilon_j\}}$ and substitute the bargaining outcomes reported in Lemma 2 and Lemma 3 to obtain

$$
\begin{aligned}
V_t^I\left(a_{ti}^m, a_{ti}^s, \varepsilon_i\right) = {} & \delta\theta \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}} \frac{\left(\varepsilon_i - \varepsilon_t^*\right) y_t}{\varepsilon_t^* y_t + \phi_t^s} \phi_t^m a_{ti}^m + \delta\theta \mathbb{I}_{\{\varepsilon_i < \varepsilon_t^*\}} \left(\varepsilon_t^* - \varepsilon_i\right) y_t a_{ti}^s \\
& + \alpha\eta \int\int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \Big[-\phi_t^m \min\left\{p_t^o(\varepsilon_j) a_{tj}^s, a_{ti}^m\right\} \\
& + \left(\varepsilon_i y_t + \phi_t^s\right) \min\left\{\tfrac{a_{ti}^m}{p_t^o(\varepsilon_j)}, a_{tj}^s\right\}\Big] dF_t^I\left(\boldsymbol{a}_{tj}\right) dG\left(\varepsilon_j\right) \\
& + \alpha\left(1-\eta\right) \int\int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} \Big[\phi_t^m \min\left\{p_t^o(\varepsilon_j) a_{ti}^s, a_{tj}^m\right\} \\
& - \left(\varepsilon_i y_t + \phi_t^s\right) \min\left\{\tfrac{a_{tj}^m}{p_t^o(\varepsilon_j)}, a_{ti}^s\right\}\Big] dF_t^I\left(\boldsymbol{a}_{tj}\right) dG\left(\varepsilon_j\right) \\
& + \phi_t^m a_{ti}^m + \left(\varepsilon_i y_t + \phi_t^s\right) a_{ti}^s + W_t^I\left(\mathbf{0}\right).
\end{aligned} \tag{107}
$$

From (11), we anticipate that as in Lagos and Wright (2005), the beginning-of-period distribution of assets across investors will be degenerate, i.e., $(a_{t+1j}^m, a_{t+1j}^s) = (A_{It+1}^m, A_{It+1}^s)$ for all $j \in \mathcal{I}$, so (107) can be written as (18). ■

Proof of Lemma 5. With Lemma 4, the dealer's problem in the second subperiod of period t, (14), becomes

$$
W_t^D\left(\mathbf{0}\right) = \max_{\tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2} \left[(-\phi_t^m + \beta \mathbb{E}_t \bar{\phi}_{t+1}) \tilde{a}_{t+1}^m + (-\phi_t^s + \beta\pi \mathbb{E}_t \bar{\phi}_{t+1} p_{t+1}) \tilde{a}_{t+1}^s\right] + \beta \mathbb{E}_t V_{t+1}^D\left(\mathbf{0}\right). \tag{108}
$$

From (18),

$$
\begin{aligned}
\int V_{t+1}^I\left(a_{t+1}^m, a_{t+1}^s, \varepsilon_i\right) dG\left(\varepsilon_i\right) = {} & \phi_{t+1}^m a_{t+1}^m + \int \left(\varepsilon_i y_{t+1} + \phi_{t+1}^s\right) a_{t+1}^s dG\left(\varepsilon_i\right) + W_{t+1}^I\left(\mathbf{0}\right) \\
& + \delta\theta \int \mathbb{I}_{\{\varepsilon_{t+1}^* \leq \varepsilon_i\}} \frac{\left(\varepsilon_i - \varepsilon_{t+1}^*\right) y_{t+1}}{\varepsilon_{t+1}^* y_{t+1} + \phi_{t+1}^s} \phi_{t+1}^m a_{t+1}^m dG\left(\varepsilon_i\right) \\
& + \delta\theta \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_{t+1}^*\}} \left(\varepsilon_{t+1}^* - \varepsilon_i\right) y_{t+1} a_{t+1}^s dG\left(\varepsilon_i\right) \\
& + \alpha\eta \int_{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s\right] \frac{1}{y_{t+1}}} \int_{\varepsilon_j} \frac{\left(\varepsilon_i - \varepsilon_j\right) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} \phi_{t+1}^m a_{t+1}^m dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \\
& + \alpha\eta \int^{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s\right] \frac{1}{y_{t+1}}} \int_{\varepsilon_j} \left(\varepsilon_i - \varepsilon_j\right) y_{t+1} A_{It+1}^s dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \\
& + \alpha\left(1-\eta\right) \int_{\left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right] \frac{1}{y_{t+1}}} \int^{\varepsilon_j} \frac{\left(\varepsilon_j - \varepsilon_i\right) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} \phi_{t+1}^m A_{It+1}^m dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right) \\
& + \alpha\left(1-\eta\right) \int^{\left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right] \frac{1}{y_{t+1}}} \int^{\varepsilon_j} \left(\varepsilon_j - \varepsilon_i\right) y_{t+1} a_{t+1}^s dG\left(\varepsilon_i\right) dG\left(\varepsilon_j\right)
\end{aligned}
$$

so the investor's problem (11) can be written as in (63), with

$$
\begin{aligned}
W_t^I(\mathbf{0}) = \max_{\tilde{a}_{t+1}^m \in \mathbb{R}_+} \Bigg\{ & -\phi_t^m \tilde{a}_{t+1}^m + \beta \mathbb{E}_t \Bigg[\Bigg(1 + \delta\theta \int \mathbb{I}_{\{\varepsilon_{t+1}^* \leq \varepsilon_i\}} \frac{\left(\varepsilon_i - \varepsilon_{t+1}^*\right) y_{t+1}}{\varepsilon_{t+1}^* y_{t+1} + \phi_{t+1}^s} dG(\varepsilon_i) \\
& + \alpha\eta \int_{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_j} \frac{\left(\varepsilon_i - \varepsilon_j\right) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} dG(\varepsilon_i)\, dG(\varepsilon_j) \Bigg) \phi_{t+1}^m \tilde{a}_{t+1}^m \\
& + \alpha\eta \int^{\left[\frac{\phi_{t+1}^m \tilde{a}_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int_{\varepsilon_j} \left(\varepsilon_i - \varepsilon_j\right) y_{t+1} dG(\varepsilon_i)\, dG(\varepsilon_j)\, A_{It+1}^s \Bigg] \Bigg\} \\
& + \max_{\tilde{a}_{t+1}^s \in \mathbb{R}_+} \Bigg\{ -\phi_t^s \tilde{a}_{t+1}^s + \beta \mathbb{E}_t \Bigg[\Bigg(\int \left(\varepsilon_i y_{t+1} + \phi_{t+1}^s\right) dG(\varepsilon_i) \\
& + \delta\theta \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_{t+1}^*\}} \left(\varepsilon_{t+1}^* - \varepsilon_i\right) y_{t+1} dG(\varepsilon_i) \\
& + \alpha(1-\eta) \int^{\left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int^{\varepsilon_j} \left(\varepsilon_j - \varepsilon_i\right) y_{t+1} dG(\varepsilon_i)\, dG(\varepsilon_j) \Bigg) a_{t+1}^s \\
& + \alpha(1-\eta) \int_{\left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s\right]\frac{1}{y_{t+1}}} \int^{\varepsilon_j} \frac{\left(\varepsilon_j - \varepsilon_i\right) y_{t+1}}{\varepsilon_j y_{t+1} + \phi_{t+1}^s} dG(\varepsilon_i)\, dG(\varepsilon_j)\, \phi_{t+1}^m A_{It+1}^m \Bigg] \Bigg\} \\
& + T_t + \beta \mathbb{E}_t W_{t+1}^I(\mathbf{0}),
\end{aligned} \tag{109}
$$

where $a_{t+1}^s = \pi \tilde{a}_{t+1}^s + (1-\pi) A^s$. The first-order necessary and sufficient conditions for optimization of (108) are (19) and (20). The first-order necessary and sufficient conditions for optimization of (109) are (21) and (22). ■

Proof of Proposition 2. In a stationary equilibrium, the dealer's Euler equations in Lemma 5 become

$$
\mu \geq \bar{\beta}, \text{ “ = ” if } \tilde{a}_{t+1d}^m > 0
$$

$$
\phi^s \geq \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\varepsilon^*, \text{ “ = ” if } \tilde{a}_{t+1d}^s > 0.
$$

The maintained assumption $\mu > \bar{\beta}$ implies $\tilde{a}_{t+1d}^m = 0$. Similarly, in a stationary monetary equilibrium the investor's Euler equations in Lemma 5 become

$$
\begin{aligned}
\mu = \bar{\beta} \Bigg[& 1 + \delta\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon^*}{\varepsilon^* + \phi^s} dG(\varepsilon_i) \\
& + \alpha\eta \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \phi^s} dG(\varepsilon_i)\, dG(\varepsilon_j) \Bigg]
\end{aligned} \tag{110}
$$

$$\phi^s \geq \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon_i)dG(\varepsilon_i) + \alpha(1-\eta)\varphi(\varepsilon^c)\right]$$

where $\varepsilon^c \equiv Z/A_I^s - \phi^s$, $\varphi(\varepsilon) \equiv \int_{\varepsilon_L}^{\varepsilon}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j - \varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon_j)$, and the second condition holds with "=" if $\tilde{a}_{t+1i}^s > 0$. Together, the dealer's and the investor's Euler equations for equity imply

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\max\left\{\varepsilon^*, \bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon_i)dG(\varepsilon_i) + \alpha(1-\eta)\varphi(\varepsilon^c)\right\}. \tag{111}$$

As $\mu \to \bar{\beta}$, (110) implies

$$\delta\theta\int_{\varepsilon^*}^{\varepsilon_H}\frac{\varepsilon_i - \varepsilon^*}{\varepsilon^* + \phi^s}dG(\varepsilon_i) + \alpha\eta\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \phi^s}dG(\varepsilon_i)\,dG(\varepsilon_j) \to 0,$$

a condition that can only hold if $\varepsilon^* \to \varepsilon_H$ and $\varepsilon^c \to \varepsilon_H$. The fact that $\varepsilon^* \to \varepsilon_H$ means that among investors who contact dealers, only those with preference type ε_H purchase equity. The fact that $\varepsilon^c \to \varepsilon_H$ implies that in bilateral trades between investors, the investor with the higher valuation purchases all his counterparty's equity holdings (the investor who wishes to buy is never constrained by his real money balances as $\mu \to \bar{\beta}$). Finally, as $\mu \to \bar{\beta}$,

$$\phi^s \to \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\max\{\varepsilon_H, \bar{\varepsilon} + \delta\theta(\varepsilon_H - \bar{\varepsilon}) + \alpha(1-\eta)\varphi(\varepsilon_H)\} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\varepsilon_H,$$

so $\tilde{a}_{t+1d}^s = A^s$ for all t, i.e., only dealers hold equity overnight. ■

Lemma 6 *Consider $\hat{\mu}$ and $\bar{\mu}$ as defined in (24). Then $\hat{\mu} < \bar{\mu}$.*

Proof of Lemma 6. Define $\Upsilon(\zeta) : \mathbb{R} \to \mathbb{R}$ by $\Upsilon(\zeta) \equiv \bar{\beta}\left[1 + \delta\theta(1-\bar{\beta}\pi)\zeta\right]$. Let $\hat{\zeta} \equiv \frac{(1-\delta\theta)(\hat{\varepsilon}-\bar{\varepsilon})}{\delta\theta\hat{\varepsilon}}$ and $\bar{\zeta} \equiv \frac{\bar{\varepsilon}-\varepsilon_L}{\beta\pi\bar{\varepsilon}+(1-\beta\pi)\varepsilon_L}$, so that $\hat{\mu} = \Upsilon(\hat{\zeta})$ and $\bar{\mu} = \Upsilon(\bar{\zeta})$. Since Υ is strictly increasing, $\hat{\mu} < \bar{\mu}$ if and only if $\hat{\zeta} < \bar{\zeta}$. With (25) and the fact that $\bar{\varepsilon} \equiv \int_{\varepsilon_L}^{\varepsilon_H}\varepsilon dG(\varepsilon) = \varepsilon_H - \int_{\varepsilon_L}^{\varepsilon_H}G(\varepsilon)\,d\varepsilon$,

$$\hat{\zeta} = \frac{\int_{\hat{\varepsilon}}^{\varepsilon_H}[1-G(\varepsilon)]\,d\varepsilon}{\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\hat{\varepsilon}}G(\varepsilon)\,d\varepsilon},$$

so clearly,

$$\hat{\zeta} < \frac{\int_{\varepsilon_L}^{\varepsilon_H}[1-G(\varepsilon)]\,d\varepsilon}{\bar{\varepsilon}} = \frac{\bar{\varepsilon} - \varepsilon_L}{\bar{\varepsilon}} < \bar{\zeta}.$$

Hence $\hat{\mu} < \bar{\mu}$. ■

Lemma 7 *In a stationary equilibrium, the interdealer market clearing condition* $\bar{A}^s_{Dt} + \bar{A}^s_{It} = A^s_{Dt} + \delta A^s_{It}$ *is equivalent to*

$$\delta\theta\left[1 - G(\varepsilon^*)\right]\left(A^s_I + \frac{Z}{\varepsilon^* + \phi^s}\right) + \delta\left(1-\theta\right)\int_{\varepsilon^*}^{\varepsilon_H}\left[A^s_I + \frac{Z}{\varepsilon + \phi^s}\right]dG\left(\varepsilon\right) = A^s_D + \delta A^s_I. \tag{112}$$

Proof of Lemma 7. Use $\delta = \kappa v$ in $\bar{A}^s_{Dt} + \bar{A}^s_{It} = A^s_{Dt} + \delta A^s_{It}$ to obtain

$$\begin{aligned}
&\theta\int\left\{\hat{a}^s_d\left[\bar{\boldsymbol{a}}_d\left(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t\right);\boldsymbol{\psi}_t\right] + \overline{a}^s_{i^*}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)\right\}dF^D_t\left(\boldsymbol{a}_{td}\right)dF^I_t\left(\boldsymbol{a}_{ti}\right)dG\left(\varepsilon\right)\\
&+\left(1-\theta\right)\int\left\{\hat{a}^s_d\left[\bar{\boldsymbol{a}}_{d^*}\left(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t\right);\boldsymbol{\psi}_t\right] + \overline{a}^s_{i}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)\right\}dF^D_t\left(\boldsymbol{a}_{td}\right)dF^I_t\left(\boldsymbol{a}_{ti}\right)dG\left(\varepsilon\right)\\
&=\int a^s_{td}dF^D_t\left(\boldsymbol{a}_{td}\right) + \int a^s_{ti}dF^I_t\left(\boldsymbol{a}_{ti}\right) + \frac{\left(1-\kappa\right)v}{\delta}\int\left[a^s_{td} - \hat{a}^s_d(\boldsymbol{a}_{td};\boldsymbol{\psi}_t)\right]dF^D_t\left(\boldsymbol{a}_{td}\right).
\end{aligned} \tag{113}$$

Since $\phi^s_t = \phi^s y_t < \varepsilon^* y_t + \phi^s y_t = \bar{\phi}^s y_t \equiv p_t\phi^m_t$ in a stationary equilibrium, Lemma 1 implies

$$\hat{a}^s_d\left[\bar{\boldsymbol{a}}_d\left(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t\right);\boldsymbol{\psi}_t\right] = \hat{a}^s_d\left[\bar{\boldsymbol{a}}_{d^*}\left(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t\right);\boldsymbol{\psi}_t\right] = \hat{a}^s_d(\boldsymbol{a}_{td};\boldsymbol{\psi}_t) = 0. \tag{114}$$

With (114) and the fact that $\int a^s_{ti}dF^I_t\left(\boldsymbol{a}_{ti}\right) = A^s_I$ and $v\int a^s_{td}dF^D_t\left(\boldsymbol{a}_{td}\right) = A^s_D$, (113) becomes

$$\begin{aligned}
A^s_D + \delta A^s_I = \delta\theta&\int\overline{a}^s_{i^*}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)dF^D_t\left(\boldsymbol{a}_{td}\right)dF^I_t\left(\boldsymbol{a}_{ti}\right)dG\left(\varepsilon\right)\\
&+\delta\left(1-\theta\right)\int\overline{a}^s_{i}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t)dF^D_t\left(\boldsymbol{a}_{td}\right)dF^I_t\left(\boldsymbol{a}_{ti}\right)dG\left(\varepsilon\right).
\end{aligned} \tag{115}$$

From Lemma 2,

$$\begin{aligned}
\overline{a}^s_{i^*}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t) &= \mathbb{I}_{\{\varepsilon^*<\varepsilon\}}\left(a^s_{ti} + \frac{1}{p_t}a^m_{ti}\right) + \mathbb{I}_{\{\varepsilon=\varepsilon^*\}}\overline{a}^s_{i^*}\\
\overline{a}^s_{i}(\boldsymbol{a}_{ti},\boldsymbol{a}_{td},\varepsilon;\boldsymbol{\psi}_t) &= \mathbb{I}_{\{\varepsilon^*<\varepsilon\}}\left[a^s_{ti} + \left(\frac{\varepsilon^*+\phi^s}{\varepsilon+\phi^s}\right)\frac{1}{p_t}a^m_{ti}\right] + \mathbb{I}_{\{\varepsilon=\varepsilon^*\}}\overline{a}^s_{i}
\end{aligned}$$

where $\overline{a}^s_{i^*}, \overline{a}^s_i \in [0, a^s_{ti} + a^m_{ti}/p_t]$, so (115) becomes

$$\delta\theta\left[1 - G(\varepsilon^*)\right]\left(A^s_I + \frac{1}{p_t}A^m_{It}\right) + \delta\left(1-\theta\right)\int_{\varepsilon^*}^{\varepsilon_H}\left[A^s_I + \left(\frac{\varepsilon^*+\phi^s}{\varepsilon+\phi^s}\right)\frac{1}{p_t}A^m_{It}\right]dG\left(\varepsilon\right) = A^s_D + \delta A^s_I.$$

Finally, use the fact that in a stationary equilibrium, $\phi^m_t A^m_{It} = Zy_t$ and $p_t\phi^m_t = \bar{\phi}^s y_t = \left(\varepsilon^* + \phi^s\right)y_t$, to arrive at the expression in the statement of the lemma. ■

Proof of Proposition 3. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. The first-order conditions for a dealer d and an investor i in the time-t Walrasian market are

$$\phi_t^s \geq \beta\pi\mathbb{E}_t\phi_{t+1}^s, \text{ “ = ” if } \tilde{a}_{t+1d}^s > 0$$
$$\phi_t^s \geq \beta\pi\mathbb{E}_t\left(\bar{\varepsilon}y_{t+1} + \phi_{t+1}^s\right), \text{ “ = ” if } \tilde{a}_{t+1i}^s > 0.$$

In a stationary equilibrium, $\mathbb{E}_t(\phi_{t+1}^s/\phi_t^s) = \bar{\gamma}$, and $\beta\bar{\gamma}\pi < 1$ is a maintained assumption, so no dealer holds equity. The Walrasian market for equity can only clear if $\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}$. This establishes parts (i) and (iii) in the statement of the proposition.

Next, we turn to monetary equilibria. With $\alpha = 0$, in a stationary equilibrium (19)-(22) become

$$\mu \geq \bar{\beta}, \text{ “ = ” if } \tilde{a}_{t+1d}^m > 0 \tag{116}$$
$$\phi^s \geq \bar{\beta}\pi\bar{\phi}^s, \text{ “ = ” if } \tilde{a}_{t+1d}^s > 0 \tag{117}$$
$$1 \geq \frac{\bar{\beta}}{\mu}\left[1 + \delta\theta\frac{\int_{\varepsilon^*}^{\varepsilon_H}\left[1 - G\left(\varepsilon\right)\right]d\varepsilon}{\varepsilon^* + \phi^s}\right], \text{ “ = ” if } \tilde{a}_{t+1i}^m > 0 \tag{118}$$
$$\phi^s \geq \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G\left(\varepsilon\right)d\varepsilon\right], \text{ “ = ” if } \tilde{a}_{t+1i}^s > 0. \tag{119}$$

(In (116) we have used the fact that $\bar{\phi}^s = \varepsilon^* + \phi^s > \phi^s$.) Under our maintained assumption $\bar{\beta} < \mu$, (116) implies $\tilde{a}_{t+1d}^m = Z_D = 0$, so (118) must hold with equality for some investor in a monetary equilibrium. Thus, in order to find a monetary equilibrium there are three possible equilibrium configurations to consider depending on the binding patterns of the complementary slackness conditions (117) and (119). The market-clearing condition, $\bar{A}_{Dt}^s + \bar{A}_{It}^s = A_{Dt}^s + \delta A_{It}^s$ must hold for all three configurations. Lemma 7 shows that this condition can be written as (112) and this condition can be rearranged to deliver (31). The rest of the proof proceeds in three steps.

Step 1: Try to construct a stationary monetary equilibrium with $\tilde{a}_{t+1d}^s = 0$ for all $d \in \mathcal{D}$, and $\tilde{a}_{t+1i}^s > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions for this case are (112) together with

$$\phi^s > \bar{\beta}\pi\bar{\phi}^s \tag{120}$$
$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \delta\theta\frac{\int_{\varepsilon^*}^{\varepsilon_H}\left[1 - G\left(\varepsilon\right)\right]d\varepsilon}{\varepsilon^* + \phi^s}\right] \tag{121}$$
$$\phi^s = \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G\left(\varepsilon\right)d\varepsilon\right] \tag{122}$$

and

$$\tilde{a}^{m}_{t+1d} = 0 \text{ for all } d \in \mathcal{D} \tag{123}$$

$$\tilde{a}^{m}_{t+1i} \geq 0, \text{ with `` > '' for some } i \in \mathcal{I} \tag{124}$$

$$\tilde{a}^{s}_{t+1d} = 0 \text{ for all } d \in \mathcal{D} \tag{125}$$

$$\tilde{a}^{s}_{t+1i} \geq 0, \text{ with `` > '' for some } i \in \mathcal{I}. \tag{126}$$

Conditions (121) and (122) are to be solved for the two unknowns ε^* and ϕ^s. Substitute (122) into (121) to obtain

$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \delta\theta \frac{\int_{\varepsilon^*}^{\varepsilon_H}\left[1 - G(\varepsilon)\right] d\varepsilon}{\varepsilon^* + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon\right]}\right] \tag{127}$$

which is a single equation in ε^*. Define

$$T(x) \equiv \frac{\int_{x}^{\varepsilon_H}\left[1 - G(\varepsilon)\right] d\varepsilon}{\frac{1}{1-\bar{\beta}\pi}x + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\hat{T}(x)} - \frac{\mu - \bar{\beta}}{\bar{\beta}\delta\theta} \tag{128}$$

with

$$\hat{T}(x) \equiv \bar{\varepsilon} - x + \delta\theta \int_{\varepsilon_L}^{x} G(\varepsilon)\, d\varepsilon, \tag{129}$$

and notice that ε^* solves (127) if and only if it satisfies $T(\varepsilon^*) = 0$. T is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$T(\varepsilon_L) = \frac{\bar{\varepsilon} - \varepsilon_L}{\varepsilon_L + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}} - \frac{\mu - \bar{\beta}}{\bar{\beta}\delta\theta},$$

$$T(\varepsilon_H) = -\frac{\mu - \bar{\beta}}{\bar{\beta}\delta\theta} < 0,$$

and

$$T'(x) = -\frac{\left[1-G(x)\right]\left\{x + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{x} G(\varepsilon)d\varepsilon\right]\right\} + \left[\int_{x}^{\varepsilon_H}\left[1-G(\varepsilon)\right]d\varepsilon\right]\left\{1 + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\delta\theta G(x)\right\}}{\left\{x + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{x} G(\varepsilon)d\varepsilon\right]\right\}^2} < 0.$$

Hence if $T(\varepsilon_L) > 0$, or equivalently, if $\mu < \bar{\mu}$ (with $\bar{\mu}$ is as defined in (24)) then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $T(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\mu \uparrow \bar{\mu}$). Once we know ε^*, ϕ^s is given by (122). Given ε^* and ϕ^s, the values of Z, $\bar{\phi}^s$, ϕ^m_t and p_t are obtained using (31) (with $A^s_I = A^s$ and $A^s_D = 0$), (28), (29) and (30). To conclude this step, notice that for this case to be an equilibrium (120) must hold, or equivalently, using (**??**) and (122), it must be that $\hat{T}(\varepsilon^*) > 0$, where $\hat{T}$ is the continuous function on $[\varepsilon_L, \varepsilon_H]$ defined in (129). Notice that $\hat{T}'(x) = -[1 - \delta\theta G(x)] < 0$, and $\hat{T}(\varepsilon_H) = -(1 - \delta\theta)(\varepsilon_H - \bar{\varepsilon}) < 0 < \bar{\varepsilon} - \varepsilon_L = \hat{T}(\varepsilon_L)$, so there exists a unique $\hat{\varepsilon} \in (\varepsilon_L, \varepsilon_H)$ such that $\hat{T}(\hat{\varepsilon}) = 0$. (Since $\hat{T}(\bar{\varepsilon}) > 0$, and $\hat{T}' < 0$, it follows that $\bar{\varepsilon} < \hat{\varepsilon}$.) Then $\hat{T}'(x) < 0$ implies $\hat{T}(\varepsilon^*) \geq 0$ if and only if $\varepsilon^* \leq \hat{\varepsilon}$, with "=" for $\varepsilon^* = \hat{\varepsilon}$. With (128), we know that $\varepsilon^* < \hat{\varepsilon}$ if and only if $T(\hat{\varepsilon}) < 0 = T(\varepsilon^*)$, i.e., if and only if

$$\bar{\beta}\left[1 + \frac{\delta\theta\left(1 - \bar{\beta}\pi\right)\int_{\hat{\varepsilon}}^{\varepsilon_H}[1 - G(\varepsilon)]\,d\varepsilon}{\bar{\beta}\pi\bar{\varepsilon} + \left(1 - \bar{\beta}\pi\right)\hat{\varepsilon} + \bar{\beta}\pi\delta\theta\int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon)\,d\varepsilon}\right] < \mu.$$

Since $\hat{T}(\hat{\varepsilon}) = (1 - \delta\theta)(\bar{\varepsilon} - \hat{\varepsilon}) + \delta\theta\int_{\hat{\varepsilon}}^{\varepsilon_H}[1 - G(\varepsilon)]\,d\varepsilon = 0$, this last condition is equivalent to $\hat{\mu} < \mu$, where $\hat{\mu}$ is as defined in (24). The allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\mu \in (\hat{\mu}, \bar{\mu})$.

Step 2: Try to construct a stationary monetary equilibrium with $a^s_{t+1d} > 0$ for some $d \in \mathcal{D}$, and $a^s_{t+1i} = 0$ for all $i \in \mathcal{I}$. The equilibrium conditions are (112), (121), (123), (124), together with

$$\phi^s = \bar{\beta}\pi\bar{\phi}^s \tag{130}$$

$$\phi^s > \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi}\left[\bar{\varepsilon} + \delta\theta\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\,d\varepsilon\right], \text{ “ = ” if } \tilde{a}^s_{t+1i} > 0. \tag{131}$$

$$\tilde{a}^s_{t+1d} \geq 0, \text{ with “ > ” for some } d \in \mathcal{D} \tag{132}$$

$$\tilde{a}^s_{t+1i} = 0, \text{ for all } i \in \mathcal{I}. \tag{133}$$

The conditions (121) and (130) are to be solved for ε^* and ϕ^s. First use $\bar{\phi}^s = \varepsilon^* + \phi^s$ in (130) to obtain

$$\phi^s = \frac{\bar{\beta}\pi}{1 - \bar{\beta}\pi}\varepsilon^*. \tag{134}$$

Substitute (134) in (121) to obtain

$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \frac{\delta\theta\left(1 - \bar{\beta}\pi\right)\int_{\varepsilon^*}^{\varepsilon_H}[1 - G(\varepsilon)]\,d\varepsilon}{\varepsilon^*}\right] \tag{135}$$

which is a single equation in ε^*. Define

$$R(x) \equiv \frac{\left(1 - \bar{\beta}\pi\right)\int_{x}^{\varepsilon_H}[1 - G(\varepsilon)]\,d\varepsilon}{x} - \frac{\mu - \bar{\beta}}{\bar{\beta}\delta\theta} \tag{136}$$

and notice that ε^* solves (135) if and only if it satisfies $R(\varepsilon^*) = 0$. R is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$R(\varepsilon_L) = \frac{\left(1-\bar{\beta}\pi\right)\left(\bar{\varepsilon}-\varepsilon_L\right)}{\varepsilon_L} - \frac{\mu-\bar{\beta}}{\bar{\beta}\delta\theta}$$
$$R(\varepsilon_H) = -\frac{\mu-\bar{\beta}}{\bar{\beta}\delta\theta}$$

and

$$R'(x) = -\frac{\left[1-G(x)\right]x + \int_x^{\varepsilon_H}\left[1-G(\varepsilon)\right]d\varepsilon}{\frac{1}{1-\bar{\beta}\pi}x^2} < 0.$$

Hence if $R(\varepsilon_L) > 0$, or equivalently, if

$$\mu < \bar{\beta}\left[1+\frac{\delta\theta\left(1-\bar{\beta}\pi\right)\left(\bar{\varepsilon}-\varepsilon_L\right)}{\varepsilon_L}\right] \equiv \mu^o$$

then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $R(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\mu \uparrow \mu^o$). Having solved for ε^*, ϕ^s is obtained from (134). Given ε^* and ϕ^s, the values of Z, $\bar{\phi}^s$, ϕ_t^m and p_t are obtained using (31) (with $A_D^s = A^s - A_I^s = \pi A^s$), (28), (29) and (30). Notice that for this case to be an equilibrium (131) must hold, or equivalently, using (134), it must be that $\hat{T}(\varepsilon^*) < 0$, which is in turn equivalent to $\hat{\varepsilon} < \varepsilon^*$. With (136), we know that $\hat{\varepsilon} < \varepsilon^*$ if and only if $R(\varepsilon^*) = 0 < R(\hat{\varepsilon})$, i.e., if and only if

$$\mu < \bar{\beta}\left[1+\frac{\delta\theta\left(1-\bar{\beta}\pi\right)\int_{\hat{\varepsilon}}^{\varepsilon_H}\left[1-G(\varepsilon)\right]d\varepsilon}{\hat{\varepsilon}}\right],$$

which using the fact that $\hat{T}(\hat{\varepsilon}) = 0$, can be written as $\mu < \hat{\mu}$. To summarize, the prices and allocations constructed in this step constitute a stationary monetary equilibrium provided $\mu \in (\bar{\beta}, \min(\hat{\mu}, \mu^o))$. To conclude this step, we show that $\hat{\mu} < \bar{\mu} < \mu^o$, which together with the previous step will mean that there is no stationary monetary equilibrium for $\mu \geq \bar{\mu}$ (thus establishing part (*ii*) in the statement of the proposition). It is clear that $\bar{\mu} < \mu^o$, and we know that $\hat{\mu} < \bar{\mu}$ from Lemma 6. Therefore the allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\mu \in (\bar{\beta}, \min(\hat{\mu}, \mu^o)) = (\bar{\beta}, \hat{\mu})$.

Step 3: Try to construct a stationary monetary equilibrium with $\tilde{a}_{t+1d}^s > 0$ for some $d \in \mathcal{D}$, and $\tilde{a}_{t+1i}^s > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions are (112), (121), (122), (123), (124), and (130) with

$$\tilde{a}_{t+1i}^s \geq 0 \text{ and } \tilde{a}_{t+1d}^s \geq 0, \text{ with " > " for some } i \in \mathcal{I} \text{ or some } d \in \mathcal{I}.$$

Notice that ε^* and ϕ^s are obtained as in Step 2. Now, however, (122) must also hold, which together with (134) implies that

$$0 = \bar{\varepsilon} - \varepsilon^* + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon$$

or equivalently, (since the right side is just $\hat{T}(\varepsilon^*)$), that $\varepsilon^* = \hat{\varepsilon}$. In other words, this condition requires $R(\hat{\varepsilon}) = \hat{T}(\hat{\varepsilon})$, or equivalently, we must have $\mu = \hat{\mu}$. As before, the market-clearing condition (31) is used to obtain Z, while (28), (29), and (30) imply $\bar{\phi}^s$, ϕ_t^m, and p_t, respectively. The allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\mu = \hat{\mu}$.

Combined, Steps 1-3 prove part (iv) in the statement of the proposition. Part $(v)(a)$ is immediate from (122) and (128), and part $(v)(b)$ from (134) and (136). ■

Corollary 2 *The marginal type, ε^*, characterized in Proposition 3 is strictly decreasing in the rate of inflation, i.e.,$\frac{\partial \varepsilon^*}{\partial \mu} < 0$ both for $\mu \in (\bar{\beta}, \hat{\mu})$, and for $\mu \in (\hat{\mu}, \bar{\mu})$.*

Proof of Corollary 2. For $\mu \in \left(\bar{\beta}, \hat{\mu}\right)$, implicitly differentiate $R(\varepsilon^*) = 0$ (with R given by (136)), and for $\mu \in (\hat{\mu}, \bar{\mu})$, implicitly differentiate $T(\varepsilon^*) = 0$ (with T given by (128)) to obtain

$$\frac{\partial \varepsilon^*}{\partial \mu} = \begin{cases} -\frac{\varepsilon^*}{\bar{\beta}\delta\theta\left(1-\bar{\beta}\pi\right)[1-G(\varepsilon^*)]+\mu-\bar{\beta}} & \text{if } \bar{\beta} < \mu < \hat{\mu} \\ -\frac{\bar{\beta}\delta\theta\int_{\varepsilon^*}^{\varepsilon_H}[1-G(\varepsilon)]d\varepsilon}{\left\{1+\bar{\beta}\delta\theta\left[\frac{\pi G(\varepsilon^*)}{1-\bar{\beta}\pi}+\frac{1-G(\varepsilon^*)}{\mu-\bar{\beta}}\right]\right\}\left(\mu-\bar{\beta}\right)^2} & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

Clearly, $\partial\varepsilon^*/\partial\mu < 0$ for $\mu \in (\bar{\beta}, \hat{\mu})$, and for $\mu \in (\hat{\mu}, \bar{\mu})$. ■

Proof of Proposition 4. With $\delta = 0$, in any stationary equilibrium the Euler equations for a dealer d obtained in Lemma 5 reduce to

$$\mu \geq \bar{\beta}, \text{ with “ = ” if } \tilde{a}_{t+1d}^m > 0$$
$$\phi^s \geq \bar{\beta}\pi\phi^s, \text{ with “ = ” if } \tilde{a}_{t+1d}^m > 0.$$

The maintained assumptions $\mu > \bar{\beta}$ and $\bar{\beta}\pi < 1$, and the fact that the equity will be valued in any equilibrium imply $\tilde{a}_{t+1d}^m = \tilde{a}_{t+1d}^m = 0$ for all $d \in \mathcal{D}$. Since dealers are inactive in any stationary equilibrium, we focus on investors for the remainder of the proof. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. The first-order condition for an investor i in the time-t Walrasian market is

$$\phi_t^s \geq \beta\pi\mathbb{E}_t\left(\bar{\varepsilon}y_{t+1} + \phi_{t+1}^s\right), \text{ “ = ” if } \tilde{a}_{t+1i}^s > 0.$$

In a stationary equilibrium the Walrasian market for equity can only clear if $\phi_t^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}y_t$. This establishes parts (i) and (iii) in the statement of the proposition. In a stationary monetary equilibrium the Euler equations for an investor obtained in Lemma 5 reduce to

$$\mu = \bar{\beta}\left[1 + \alpha\eta \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \phi^s} dG(\varepsilon_i)\, dG(\varepsilon_j)\right] \tag{137}$$

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\int_{\varepsilon_L}^{\varepsilon^c}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j - \varepsilon_i)\, dG(\varepsilon_i)\, dG(\varepsilon_j)\right] \tag{138}$$

where

$$\varepsilon^c \equiv \frac{Z}{A^s} - \phi^s. \tag{139}$$

Condition (137) can be substituted into (138) to obtain a single equation in the unknown ε^c, namely $\bar{T}(\varepsilon^c) = 0$, where $\bar{T} : [\varepsilon_L, \varepsilon_H] \to \mathbb{R}$ is defined by

$$\bar{T}(\varepsilon^c) \equiv \bar{\beta}\alpha\eta \int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\bar{\varepsilon} + \alpha(1-\eta)\int_{\varepsilon_L}^{\varepsilon^c}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j)\right]} dG(\varepsilon_i)\, dG(\varepsilon_j) + \bar{\beta} - \mu.$$

Notice that $\bar{T}(\varepsilon_H) = \bar{\beta} - \mu < 0$ and

$$\bar{T}(\varepsilon_L) = \bar{\beta}\alpha\eta \int_{\varepsilon_L}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\bar{\varepsilon}} dG(\varepsilon_i)\, dG(\varepsilon_j) + \bar{\beta} - \mu,$$

so since $\bar{T}$ is continuous, a stationary monetary equilibrium exists if $\mu < \tilde{\mu}$ with $\tilde{\mu}$ defined as in (35). In addition,

$$\bar{T}'(\varepsilon^c) = -\Bigg[\bar{\beta}\alpha\eta \int_{\varepsilon^c}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon^c}{\varepsilon^c + \phi^s} dG(\varepsilon_i)\, G'(\varepsilon^c)$$
$$+ \frac{(\bar{\beta}\alpha)^2\pi\eta(1-\eta)}{1-\bar{\beta}\pi}\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j)\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c - \varepsilon) dG(\varepsilon) G'(\varepsilon^c)}{(\varepsilon_j + \phi^s)^2} dG(\varepsilon_i)\, dG(\varepsilon_j)\Bigg]$$

is negative, so a stationary monetary equilibrium exists if and only if $\mu < \tilde{\mu}$, and there cannot be more than one stationary monetary equilibrium. Condition (36) is just (138), condition (38) is $\bar{T}(\varepsilon^c) = 0$, and (37) follows from (139). This establishes parts (ii) and (iv). Part (v) is immediate from (38). ■

Proof of Proposition 5. Recall that $\partial\varepsilon^*/\partial\mu < 0$ (Corollary 2). (i) From (27),

$$\frac{\partial \phi^s}{\partial \mu} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\mathbb{I}_{\{\bar{\beta}<\mu\leq\hat{\mu}\}} + \mathbb{I}_{\{\hat{\mu}<\mu<\bar{\mu}\}}\delta\theta G(\varepsilon^*)\right]\frac{\partial \varepsilon^*}{\partial \mu} < 0.$$

(ii) Condition (28) implies $\partial\bar{\phi}^s/\partial\mu = \partial\varepsilon^*/\partial\mu + \partial\phi^s/\partial\mu < 0$. (iii) Differentiate (31) to obtain

$$\frac{\partial Z}{\partial \varepsilon^*} = \delta Z \frac{G'(\varepsilon^*)A_I^s + \left[G'(\varepsilon^*)(\varepsilon^*+\phi^s)+\theta[1-G(\varepsilon^*)]\left(1+\frac{\partial\phi^s}{\partial\varepsilon^*}\right)+(1-\theta)\frac{\partial\phi^s}{\partial\varepsilon^*}\int_{\varepsilon^*}^{\varepsilon_H}\left(\frac{\varepsilon^*+\phi^s}{\varepsilon+\phi^s}\right)^2 dG(\varepsilon)\right]\frac{Z}{(\varepsilon^*+\phi^s)^2}}{A_D^s+\delta G(\varepsilon^*)A_I^s} > 0. \tag{140}$$

Hence $\partial Z/\partial\mu = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial\mu) < 0$. From (29), $\partial\phi_t^m/\partial\mu = (y_t/A_t^m)\,\partial Z/\partial\mu < 0$. ■

Proof of Proposition 6. First, notice that $\partial\varepsilon^c/\partial\mu = 1/\bar{T}'(\varepsilon^c) < 0$, where $\bar{T}(\cdot)$ is the mapping defined in the proof of Proposition 4. (i) Differentiate (36) to obtain

$$\frac{\partial\phi^s}{\partial\mu} = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\alpha(1-\eta)\,G'(\varepsilon^c)\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)\,dG(\varepsilon_i)\,\frac{\partial\varepsilon^c}{\partial\mu} < 0.$$

(ii) From (37), $\partial Z/\partial\mu = (\partial\varepsilon^c/\partial\mu + \partial\phi^s/\partial\mu)A^s < 0$, and since $Z = \phi_t^m A_t^m/y_t$, $\partial\phi_t^m/\partial\mu = (\partial Z/\partial\mu)\,(y_t/A_t^m) < 0$. ■

Proof of Proposition 7. From condition (32),

$$\frac{\partial\varepsilon^*}{\partial(\delta\theta)} = \frac{\frac{\mu-\bar{\beta}}{\delta\theta}\left[\varepsilon^* + \bar{\beta}\pi(\bar{\varepsilon}-\varepsilon^*)\,\mathbb{I}_{\{\hat{\mu}<\mu\}}\right]}{\bar{\beta}\delta\theta(1-\bar{\beta}\pi)\left[1-G(\varepsilon^*)\right] + (\mu-\bar{\beta})\left\{1+\bar{\beta}\pi\left[\delta\theta G(\varepsilon^*)-1\right]\mathbb{I}_{\{\hat{\mu}<\mu\}}\right\}} > 0. \tag{141}$$

(i) From (36),

$$\frac{\partial\phi^s}{\partial(\delta\theta)} = \begin{cases} \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\frac{\partial\varepsilon^*}{\partial(\delta\theta)} > 0 & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\left[\int_{\varepsilon_L}^{\varepsilon^*}G(\varepsilon)\,d\varepsilon + \delta\theta G(\varepsilon^*)\frac{\partial\varepsilon^*}{\partial(\delta\theta)}\right] > 0 & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

(ii) From (28), $\partial\bar{\phi}^s/\partial(\delta\theta) = \partial\varepsilon^*/\partial(\delta\theta) + \partial\phi^s/\partial(\delta\theta) > 0$. (iii) For $\mu \in (\hat{\mu},\bar{\mu})$, (31) implies $\partial Z/\partial\delta = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial\delta) > 0$ (the sign follows from (140) and (141)), and therefore $\partial\phi_t^m/\partial\delta = (\partial Z/\partial\delta)\,(y_t/A_t^m) > 0$. ■

Proof of Proposition 8. Implicit differentiation of $\bar{T}(\varepsilon^c) = 0$ implies

$$\frac{\partial\varepsilon^c}{\partial\alpha} = \frac{\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\eta(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)\left[(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi\bar{\varepsilon}\right]}{\left\{(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\right\}^2}dG(\varepsilon_i)dG(\varepsilon_j)}{\int_{\varepsilon^c}^{\varepsilon_H}\frac{\alpha\eta(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)}{(1-\bar{\beta}\pi)\varepsilon^c+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]}dG(\varepsilon_i)G'(\varepsilon^c)+\int_{\varepsilon^c}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\bar{\beta}\pi\alpha^2\eta(1-\eta)(1-\bar{\beta}\pi)(\varepsilon_i-\varepsilon_j)\varphi'(\varepsilon^c)}{\left\{(1-\bar{\beta}\pi)\varepsilon_j+\bar{\beta}\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\right\}^2}dG(\varepsilon_i)dG(\varepsilon_j)} > 0.$$

(i) Differentiate (36) to arrive at

$$\frac{\partial\phi^s}{\partial\alpha} = \frac{\bar{\beta}\pi(1-\eta)}{1-\bar{\beta}\pi}\left[\varphi(\varepsilon^c) + \alpha\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon^c)\,\frac{\partial\varepsilon^c}{\partial\alpha}\right] > 0.$$

(ii) From (37),

$$\frac{\partial Z}{\partial \alpha} = \left(\frac{\partial \varepsilon^c}{\partial \alpha} + \frac{\partial \phi^s}{\partial \alpha} \right) A^s > 0,$$

and since $Z = \phi_t^m A_t^m / y_t$, it follows that $\partial \phi_t^m / \partial \alpha > 0$. ■

Proof of Proposition 9. (i) The result is immediate from the expression for A_D^s in Proposition 3. (ii) From (24) and (25),

$$\frac{\partial \hat{\mu}}{\partial (\delta\theta)} = \bar{\beta} \left(1 - \bar{\beta}\pi\right) \left\{ \frac{(1-\delta\theta)\,\bar{\varepsilon}}{\left[1 - \delta\theta G(\hat{\varepsilon})\right] \hat{\varepsilon}^2} \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon)\, d\varepsilon - \frac{\hat{\varepsilon} - \bar{\varepsilon}}{\hat{\varepsilon}} \right\}.$$

Notice that $\partial \hat{\mu} / \partial (\delta\theta)$ approaches a positive value as $\delta\theta \to 0$, and a negative value as $\delta\theta \to 1$. Also, $\hat{\mu} \to \bar{\beta}$ both when $\delta\theta \to 0$, and when $\delta\theta \to 1$. Hence $\mu > \bar{\beta} = \lim_{\delta\theta \to 0} \hat{\mu} = \lim_{\delta\theta \to 1} \hat{\mu}$ for a range of values of $\delta\theta$ close to 0 and a range of values of $\delta\theta$ close to 1. For those ranges of values of $\delta\theta$, $A_D^s = 0$. In between those ranges there must exist values of $\delta\theta$ such that $\mu < \hat{\mu}$ which implies $A_D^s > 0$. ■

Proof of Proposition 10. (i) Differentiate (39) to get

$$\frac{\partial \mathcal{V}}{\partial \mu} = 2\delta G'(\varepsilon^*)\,(A^s - \pi \tilde{A}_D^s) \frac{\partial \varepsilon^*}{\partial \mu} < 0,$$

where the inequality follows from Corollary 2. (ii) From (39),

$$\frac{\partial \mathcal{V}}{\partial \theta} = 2\delta G'(\varepsilon^*)\,(A^s - \pi \tilde{A}_D^s) \frac{\partial \varepsilon^*}{\partial \theta}$$
$$\frac{\partial \mathcal{V}}{\partial \delta} = 2 \left[G(\varepsilon^*) + \delta G'(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial \delta} \right] (A^s - \pi \tilde{A}_D^s)$$

and both are positive since $\partial \varepsilon^* / \partial (\delta\theta) > 0$ (see (141)). ■

Proof of Proposition 10. Rewrite $\tilde{\mathcal{V}}$ as

$$\tilde{\mathcal{V}} = \alpha A^s \int_{\varepsilon_L}^{\varepsilon^c} \left\{ \eta \left[1 - G(\varepsilon_i)\right] + (1-\eta)\, G(\varepsilon_i) \right\} dG(\varepsilon_i)$$
$$+ \alpha A^s \int_{\varepsilon^c}^{\varepsilon_H} \left\{ \eta \left[1 - G(\varepsilon_i)\right] + (1-\eta)\, G(\varepsilon_i) \right\} \frac{\varepsilon^c + \phi^s}{\varepsilon_i + \phi^s} dG(\varepsilon_i).$$

Differentiate to obtain

$$\frac{\partial \tilde{\mathcal{V}}}{\partial \varepsilon^c} = \alpha A^s \int_{\varepsilon^c}^{\varepsilon_H} \left\{ \eta \left[1 - G(\varepsilon_i)\right] + (1-\eta)\, G(\varepsilon_i) \right\} \frac{\partial}{\partial \varepsilon^c} \left[\frac{\varepsilon^c + \phi^s}{\varepsilon_i + \phi^s} \right] dG(\varepsilon_i),$$

where

$$\frac{\partial}{\partial \varepsilon^c}\left[\frac{\varepsilon^c+\phi^s}{\varepsilon_i+\phi^s}\right]=\frac{\varepsilon_i+\phi^s+(\varepsilon_i-\varepsilon^c)\frac{\partial\phi^s}{\partial\varepsilon^c}}{(\varepsilon_i+\phi^s)^2}A^s>0 \text{ for } \varepsilon_i>\varepsilon^c.$$

Hence, $\partial\tilde{\mathcal{V}}/\partial\varepsilon^c>0$. Thus $\partial\tilde{\mathcal{V}}/\partial\mu=(\partial\tilde{\mathcal{V}}/\partial\varepsilon^c)(\partial\varepsilon^c/\partial\mu)<0$, since $\partial\varepsilon^c/\partial\mu<0$ (see proof of Proposition 6), which establishes (i). For part (ii), simply notice that $\partial\tilde{\mathcal{V}}/\partial\alpha=\tilde{\mathcal{V}}/\alpha+(\partial\tilde{\mathcal{V}}/\partial\varepsilon^c)(\partial\varepsilon^c/\partial\alpha)>0$. ■

Proof of Proposition 12. (i) For $\bar{\beta}<\mu\le\hat{\mu}$, $\partial\mathcal{P}/\partial\mu=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right](\partial\varepsilon^*/\partial\mu)<0$, and for $\hat{\mu}<\mu<\bar{\mu}$, $\partial\mathcal{P}/\partial\mu=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\delta\theta G(\varepsilon^*)(\partial\varepsilon^*/\partial\mu)<0$. (ii) For $\bar{\beta}<\mu\le\hat{\mu}$, $\partial\mathcal{P}/\partial(\delta\theta)=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right](\partial\varepsilon^*/\partial(\delta\theta))>0$, and for $\hat{\mu}<\mu<\bar{\mu}$, $\partial\mathcal{P}/\partial\mu=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\{\delta\theta G(\varepsilon^*)[\partial\varepsilon^*/\partial(\delta\theta)]+\int_{\varepsilon_L}^{\varepsilon^*}G(\varepsilon)\,d\varepsilon\}>0$. ■

Proof of Proposition 13. (i) $\partial\tilde{\mathcal{P}}/\partial\mu=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right]\alpha(1-\eta)\varphi'(\varepsilon^c)(\partial\varepsilon^c/\partial\mu)<0$. (ii) $\partial\tilde{\mathcal{P}}/\partial\alpha=\left[\bar{\beta}\pi/(1-\bar{\beta}\pi)\right](1-\eta)\{\alpha\varphi'(\varepsilon^c)(\partial\varepsilon^c/\partial\alpha)+\varphi(\varepsilon^c)\}>0$. ■

Proof of Proposition 14. The choice variable a'_{tD} does not appear in the Planner's objective function, so $a'_{tD}=0$ at an optimum. Since (42) must bind for every t at an optimum, the planner's problem is equivalent to

$$W^{**}=\max_{\{v_t,\tilde{a}_{tD},\tilde{a}_{tI},a'_{tI}\}_{t=0}^{\infty}}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\delta(v_t)\int_{[\varepsilon_L,\varepsilon_H]}\varepsilon a'_{tI}(d\varepsilon)+[1-\delta(v_t)]\bar{\varepsilon}a_{tI}-kv_{t+1}\right\}y_t$$

subject to (6), (7), (40) and (41). Clearly, $\int_{[\varepsilon_L,\varepsilon_H]}\varepsilon a'_{tI}(d\varepsilon)\le\varepsilon_H$ and (41) must bind at an optimum, so $W^{**}\le\bar{W}^{**}$, where

$$\begin{aligned}
\bar{W}^{**}&=\max_{\{v_t,\tilde{a}_{tD},\tilde{a}_{tI}\}_{t=0}^{\infty}}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\{[v_ta_{tD}+\delta(v_t)a_{tI}]\varepsilon_H+[1-\delta(v_t)]\bar{\varepsilon}a_{tI}-kv_{t+1}\}y_t\\
&=\max_{\{v_t,\tilde{a}_{tI}\}_{t=0}^{\infty}}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\{[\pi\varepsilon_H+(1-\pi)\{\delta(v_t)\varepsilon_H+[1-\delta(v_t)]\bar{\varepsilon}\}]A^s\\
&\quad-[1-\delta(v_t)](\varepsilon_H-\bar{\varepsilon})\pi\tilde{a}_{tI}-kv_{t+1}\}y_t\\
&=\max_{\{v_t\}_{t=0}^{\infty}}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\{[\pi\varepsilon_H+(1-\pi)\{\delta(v_t)\varepsilon_H+[1-\delta(v_t)]\bar{\varepsilon}\}]A^s-kv_{t+1}\}y_t\\
&=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{[\pi\varepsilon_H+(1-\pi)\{\delta(v_t^*)\varepsilon_H+[1-\delta(v_t^*)]\bar{\varepsilon}\}]A^s-kv_{t+1}^*\right\}y_t,
\end{aligned}$$

where the maximization in the first line is subject to (6), (7) and (40) (which must bind at an optimum), the second line has been obtained by substituting these constraints into the objective function, and $\{v_t^*\}$ in the last line denotes the sequence of v_t characterized by (43). The allocation in the statement of the proposition achieves $\bar{W}^{**}$ and therefore solves the Planner's problem. ■

Lemma 8 *In any equilibrium, the free-entry condition (44) can be written as (45).*

Proof of Lemma 8. With (14), the left side of condition (44) can be written as

$$\max_{(a_{t+1}^m, a_{t+1}^s)\in\mathbb{R}_+^2} \left[\beta\mathbb{E}_t V_{t+1}^D\left(a_{t+1}^m, \pi a_{t+1}^s\right) - \left(\phi_t^m a_{t+1}^m + \phi_t^s a_{t+1}^s\right)\right] - k_t.$$

And with (17), this last expression becomes

$$\max_{(a_{t+1}^m, a_{t+1}^s)\in\mathbb{R}_+^2} \left[\left(\beta\mathbb{E}_t\bar{\phi}_{t+1} - \phi_t^m\right)a_{t+1}^m + \left(\beta\pi\mathbb{E}_t\bar{\phi}_{t+1}p_{t+1} - \phi_t^s\right)a_{t+1}^s\right] + \beta\mathbb{E}_t V_{t+1}^D(\mathbf{0}) - k_t, \quad (142)$$

where

$$V_{t+1}^D(\mathbf{0}) \equiv \kappa(v_{t+1})(1-\theta)\bar{\phi}_{t+1}\left[A_{It+1}^m \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{(\varepsilon-\varepsilon_{t+1}^*)y_{t+1}}{\varepsilon y_{t+1}+\phi_{t+1}^s} dG(\varepsilon) + p_{t+1}A_{It+1}^s \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \frac{(\varepsilon_{t+1}^*-\varepsilon)y_{t+1}}{\varepsilon_{t+1}^* y_{t+1}+\phi_{t+1}^s} dG(\varepsilon)\right]$$
$$+ \max\left\{W_{t+1}^D(\mathbf{0}) - k_t, 0\right\}$$

is as in Lemma 4, except for the last term, which reflects the fact that the dealer has to bear cost k in order to participate in the OTC market of the following period. In equilibrium, the dealer optimization (conditions (19) and (20)) implies

$$\max_{(a_{t+1}^m, a_{t+1}^s)\in\mathbb{R}_+^2} \left[\left(\beta\mathbb{E}_t\bar{\phi}_{t+1} - \phi_t^m\right)a_{t+1}^m + \left(\beta\pi\mathbb{E}_t\bar{\phi}_{t+1}p_{t+1} - \phi_t^s\right)a_{t+1}^s\right] = 0.$$

Also, (44) implies $\max\left\{W_{t+1}^D(\mathbf{0}) - k_t, 0\right\} = 0$. Hence (142) reduces to $\Phi_{t+1} - k_t$, with Φ_{t+1} as defined below (45). ■

Proof of Proposition 15. Consider a stationary equilibrium with free entry (for the model with $\alpha = 0$). As $\mu \to \bar{\beta}$, (32) implies

$$\frac{\int_{\varepsilon^*}^{\varepsilon_H}[1-G(\varepsilon)]\,d\varepsilon}{\varepsilon^* + \beta\pi\left[\bar{\varepsilon} - \varepsilon^* + \delta(v)\theta\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\,d\varepsilon\right]\mathbb{I}_{\{\hat{\mu}<\mu\}}} \to 0$$

which in turn implies $\varepsilon^* \to \varepsilon_H$. The dealer's and the investor's Euler equations for equity in Lemma 5 imply

$$\phi^s = \frac{\bar{\beta}\pi}{1-\bar{\beta}\pi}\max\left\{\varepsilon^*, \bar{\varepsilon} + \delta(v)\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon)dG(\varepsilon)\right\},$$

and as $\varepsilon^* \to \varepsilon_H$, $\max\left\{\varepsilon^*, \bar{\varepsilon} + \delta(v)\,\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)\right\} \to \max\{\varepsilon_H, \bar{\varepsilon} + \delta(v)\,\theta(\varepsilon_H - \bar{\varepsilon})\} = \varepsilon_H$, so $\tilde{A}_D^s \to A^s$, i.e., only dealers hold equity overnight. Thus, from (48), $\bar{\Phi} - k \to \Pi(v)$, where

$$\Pi(v) \equiv \bar{\beta}\kappa(v)(1-\theta)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s - k.$$

Notice that

$$\lim_{v\to\infty} \Pi(v) = -k < 0 < \bar{\beta}(1-\theta)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s - k = \Pi(0)$$

and $\Pi'(v) = \bar{\beta}\kappa'(v)(1-\theta)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s < 0$, so there exists a unique $v \in (0,\infty)$ that satisfies $\Pi(v) = 0$. To conclude, we only need to show that under the hypothesis of the proposition, $\Pi(v) = 0$ is equivalent to (43). Notice that $\delta''(v) < 0$ implies $\kappa(v) = \delta(v)/v \leq \delta'(0)$ for any $v \geq 0$. In particular, for $v = 0$ this implies $1 \leq \delta'(0)$. Hence

$$0 < \bar{\beta}(1-\theta)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s - k \leq \bar{\beta}\delta'(0)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s - k$$

which means that $v > 0$ in the Planner's solution. Then (43) must hold with equality and the optimal v satisfies

$$\bar{\beta}\delta'(v)(\varepsilon_H - \bar{\varepsilon})(1-\pi) A^s - k = 0. \tag{143}$$

Finally, notice that $\delta'(v) = \kappa(v) + \kappa'(v)\,v$, so if $1-\theta = 1 - \frac{-\kappa'(v)v}{\kappa(v)} = \frac{\delta'(v)}{\kappa(v)}$ then (143) is identical to $\Pi(v) = 0$. ■

B Supplementary material

B.1 Dynamics

In this section we consider dynamic equilibria, i.e., equilibria in which asset holdings $\{A_{Dt}^s, A_{It}^s\}$ may vary over time and real asset prices are linear, but possibly time-varying functions of the aggregate dividend. Specifically, the real balances held by investors are given by $\phi_t^m A_{It}^m = Z_t y_t$ (dealers do not hold money overnight in any equilibrium), the real ex-dividend equity price is $f_t^s(y_t) = \phi_t^s y_t$, and the real cum-dividend equity price is $\bar{f}_t^s(y_t) \equiv p_t\phi_t^m = \bar{\phi}_t^s y_t$. Hence, $\varepsilon_t^* \equiv \frac{\bar{f}_t^s(y_t) - f_t^s(y_t)}{y_t} = \bar{\phi}_t^s - \phi_t^s$ may now also vary over time.

B.1.1 Pure-dealer OTC market

A dynamic equilibrium for the pure-dealer OTC market is a bounded sequence $\{Z_t, A^s_{Dt}, A^s_{It}, \phi^s_t, \varepsilon^*_t\}_{t=0}^{\infty}$ such that $\{\phi^s_t, \varepsilon^*_t\}_{t=0}^{\infty}$ satisfies the following system of difference equations

$$Z_t = \frac{\bar{\beta}}{\mu}\left[1+\delta\theta\int_{\varepsilon^*_{t+1}}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^*_{t+1}}{\varepsilon^*_{t+1}+\phi^s_{t+1}}dG(\varepsilon)\right]Z_{t+1}$$

$$\phi^s_t = \bar{\beta}\pi\left[\phi^s_{t+1}+\max\left\{\varepsilon^*_{t+1},\bar{\varepsilon}+\delta\theta\int_{\varepsilon_L}^{\varepsilon^*_{t+1}}G(\varepsilon)\,d\varepsilon\right\}\right]$$

and $\{Z_t, A^s_{Dt}, A^s_{It}\}_{t=0}^{\infty}$ is given by

$$Z_t = \frac{A^s_{Dt}+\delta G(\varepsilon^*_t)A^s_{It}}{\delta\theta\left[1-G(\varepsilon^*_t)\right]\frac{1}{\varepsilon^*_t+\phi^s_t}+\delta(1-\theta)\int_{\varepsilon^*_t}^{\varepsilon_H}\frac{1}{\varepsilon+\phi^s_t}dG(\varepsilon)}$$

$$A^s_{Dt} = \mathbb{I}_{\{\hat{\varepsilon}\leq\varepsilon^*_t\}}\pi A^s$$

$$A^s_{It} = \left[\mathbb{I}_{\{\varepsilon^*_t<\hat{\varepsilon}\}}+\mathbb{I}_{\{\hat{\varepsilon}\leq\varepsilon^*_t\}}(1-\pi)\right]A^s.$$

B.1.2 Non-intermediated OTC market

A dynamic equilibrium for the non-intermediated OTC market is a bounded sequence $\{Z_t, \phi^s_t\}_{t=0}^{\infty}$ that satisfies the following system of difference equations

$$Z_t = \frac{\bar{\beta}}{\mu}\left[1+\alpha\eta\int_{\left[\frac{Z_{t+1}}{A^s}-\phi^s_{t+1}\right]}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j+\phi^s_{t+1}}dG(\varepsilon_i)\,dG(\varepsilon_j)\right]Z_{t+1}$$

$$\phi^s_t = \bar{\beta}\pi\left[\phi^s_{t+1}+\bar{\varepsilon}+\alpha(1-\eta)\int_{\varepsilon_L}^{\left[\frac{Z_{t+1}}{A^s}-\phi^s_{t+1}\right]}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon_j)\right].$$

Equivalently, letting $\varepsilon^c_{t+1}\equiv\frac{Z_{t+1}}{A^s}-\phi^s_{t+1}$, an equilibrium for the non-intermediated OTC market is a bounded sequence $\{Z_t, \varepsilon^c_t\}_{t=0}^{\infty}$ that satisfies the following system of difference equations

$$z_t = \frac{\bar{\beta}}{\mu}\left[1+\alpha\eta\int_{\varepsilon^c_{t+1}}^{\varepsilon_H}\int_{\varepsilon_j}^{\varepsilon_H}\frac{\varepsilon_i-\varepsilon_j}{\varepsilon_j-\varepsilon^c_{t+1}+z_{t+1}}dG(\varepsilon_i)\,dG(\varepsilon_j)\right]z_{t+1}$$

$$z_t-\varepsilon^c_t = \bar{\beta}\pi\left[z_{t+1}-\varepsilon^c_{t+1}+\bar{\varepsilon}+\alpha(1-\eta)\int_{\varepsilon_L}^{\varepsilon^c_{t+1}}\int_{\varepsilon_L}^{\varepsilon_j}(\varepsilon_j-\varepsilon_i)\,dG(\varepsilon_i)\,dG(\varepsilon_j)\right].$$

where $z_t\equiv Z_t/A^s$.

B.1.3 Pure-dealer OTC market with dealer entry

A dynamic equilibrium for the pure-dealer OTC market with dealer entry is a bounded sequence $\{v_t, Z_t, A^s_{Dt}, A^s_{It}, \phi^s_t, \varepsilon^*_t\}_{t=0}^{\infty}$ such that $\{\phi^s_t, \varepsilon^*_t\}_{t=0}^{\infty}$ satisfies the following system of difference equations

$$Z_t = \frac{\bar{\beta}}{\mu}\left[1 + \delta(v_{t+1})\theta \int_{\varepsilon^*_{t+1}}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*_{t+1}}{\varepsilon^*_{t+1} + \phi^s_{t+1}} dG(\varepsilon)\right] Z_{t+1}$$

$$\phi^s_t = \bar{\beta}\pi \left[\phi^s_{t+1} + \max\left\{\varepsilon^*_{t+1}, \bar{\varepsilon} + \delta(v_{t+1})\theta \int_{\varepsilon_L}^{\varepsilon^*_{t+1}} G(\varepsilon)\, d\varepsilon\right\}\right]$$

and $\{v_t, Z_t, A^s_{Dt}, A^s_{It}\}_{t=0}^{\infty}$ is given by

$$Z_t = \frac{A^s_{Dt} + \delta(v_t)\, G(\varepsilon^*_t) A^s_{It}}{\delta(v_t)\theta\left[1 - G(\varepsilon^*_t)\right]\frac{1}{\varepsilon^*_t + \phi^s_t} + \delta(v_t)(1-\theta)\int_{\varepsilon^*_t}^{\varepsilon_H} \frac{1}{\varepsilon + \phi^s_t} dG(\varepsilon)}$$

$$A^s_{It} = \begin{cases} (1-\pi) A^s & \text{if } \bar{\varepsilon} + \delta(v_t)\theta \int_{\varepsilon_L}^{\varepsilon^*_t} G(\varepsilon)\, d\varepsilon \le \varepsilon^*_t \\ A^s & \text{if } \varepsilon^*_t < \bar{\varepsilon} + \delta(v_t)\theta \int_{\varepsilon_L}^{\varepsilon^*_t} G(\varepsilon)\, d\varepsilon \end{cases}$$

$$A^s_{Dt+1} = A^s - A^s_{It+1}$$

$$v_{t+1} \begin{cases} = 0 & \text{if } \Psi_{t+1}(0) - k \le 0 \\ \in \{v \in \mathbb{R}_+ : \Psi_{t+1}(v) = k\} & \text{if } 0 < \Psi_{t+1}(0) - k \end{cases}$$

where $\kappa(v) \equiv \delta(v)/v$ and

$$\Psi_{t+1}(v) \equiv \bar{\beta}\kappa(v)(1-\theta)\left[A^s_{It+1}\int_{\varepsilon_L}^{\varepsilon^*_{t+1}} (\varepsilon^*_{t+1} - \varepsilon) dG(\varepsilon) + Z_{t+1}\int_{\varepsilon^*_{t+1}}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*_{t+1}}{\varepsilon + \phi^s_{t+1}} dG(\varepsilon)\right].$$

B.2 Illiquid bonds

Lemma 9 *In a stationary monetary equilibrium, the nominal yield to maturity of a nominal risk-free illiquid bond of any maturity is ι as given in (23).*

Proof of Lemma 9. Let $\phi^B_{t,k}$ denote the real price (in terms of the second-subperiod consumption good of time t) of an N-period risk-free pure-discount nominal bond that matures in period $t+k$, for $k = 0, 1, 2, ..., N$ (so k is the number of periods until the bond matures). Assume that the bond is illiquid in the sense that it cannot be traded in the OTC market. For any t, the Euler equation for this asset is $\phi^B_{t,k} = \beta \mathbb{E}_t \phi^B_{t+1,k-1}$, for $k = 1, ..., N$, with $\phi^B_{t,0} = \phi^m_t$. Hence, using the Law of Iterated Expectations,

$$\phi^B_{t,k} = \beta^k \mathbb{E}_t \phi^m_{t+k} \tag{144}$$

for $k=1,...,N$. The dollar price of the bond in the second subperiod of period t is $q^B_{t,k}=\phi^B_{t,k}/\phi^m_t$ for $k=0,...,N$. Thus, (144) implies

$$q^B_{t,k}=\beta^k \mathbb{E}_t \frac{\phi^m_{t+k}}{\phi^m_t}=\left(\frac{\bar{\beta}}{\mu}\right)^k,$$

where the last equality follows from the fact that $\phi^m_{t+1}=(\gamma_{t+1}/\mu)\,\phi^m_t$ in a stationary monetary equilibrium. The nominal yield (to maturity) at period t for a nominal bond with k periods until maturity, is defined as the $\iota_{t,k}$ that solves $(1+\iota_{t,k})^k=1/q^B_{t,k}$, so

$$\iota_{t,k}=\left(q^B_{t,k}\right)^{-1/k}-1=\frac{\mu-\bar{\beta}}{\bar{\beta}}\equiv\iota$$

as defined in (23). ■

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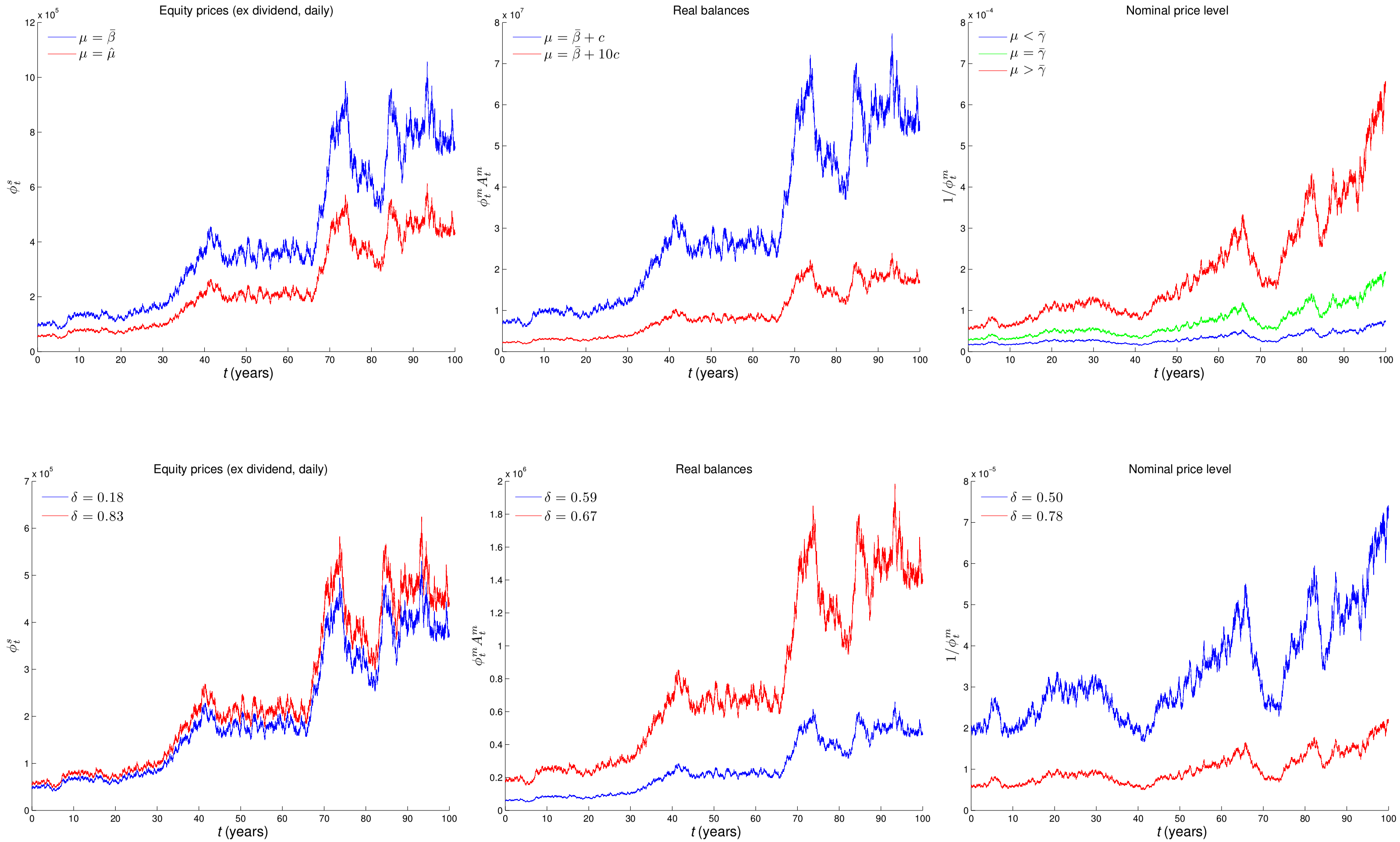

Figure 1: Effects of monetary policy and market structure on equity prices, real money balances, and the price level

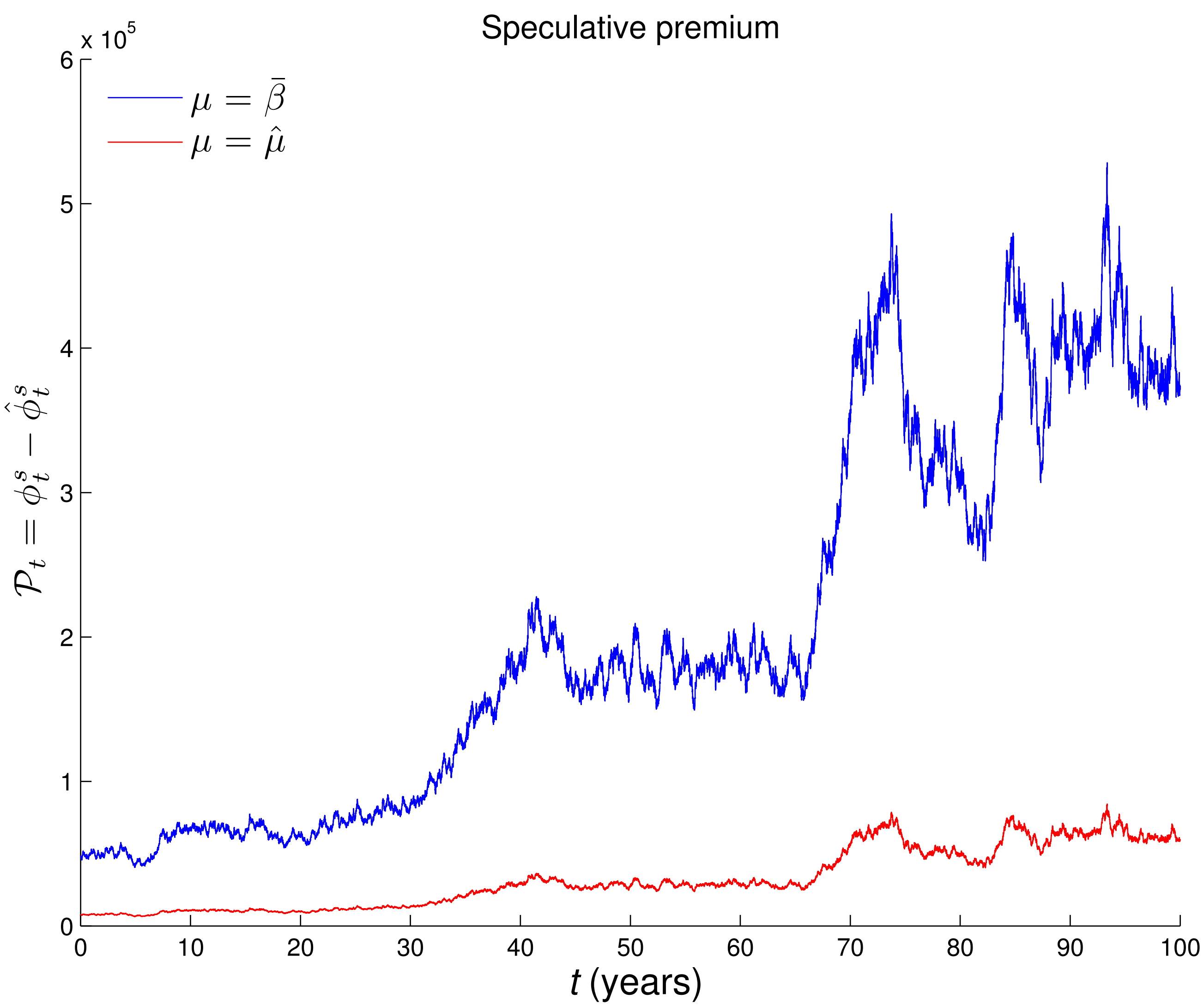

Figure 2: Speculative premium

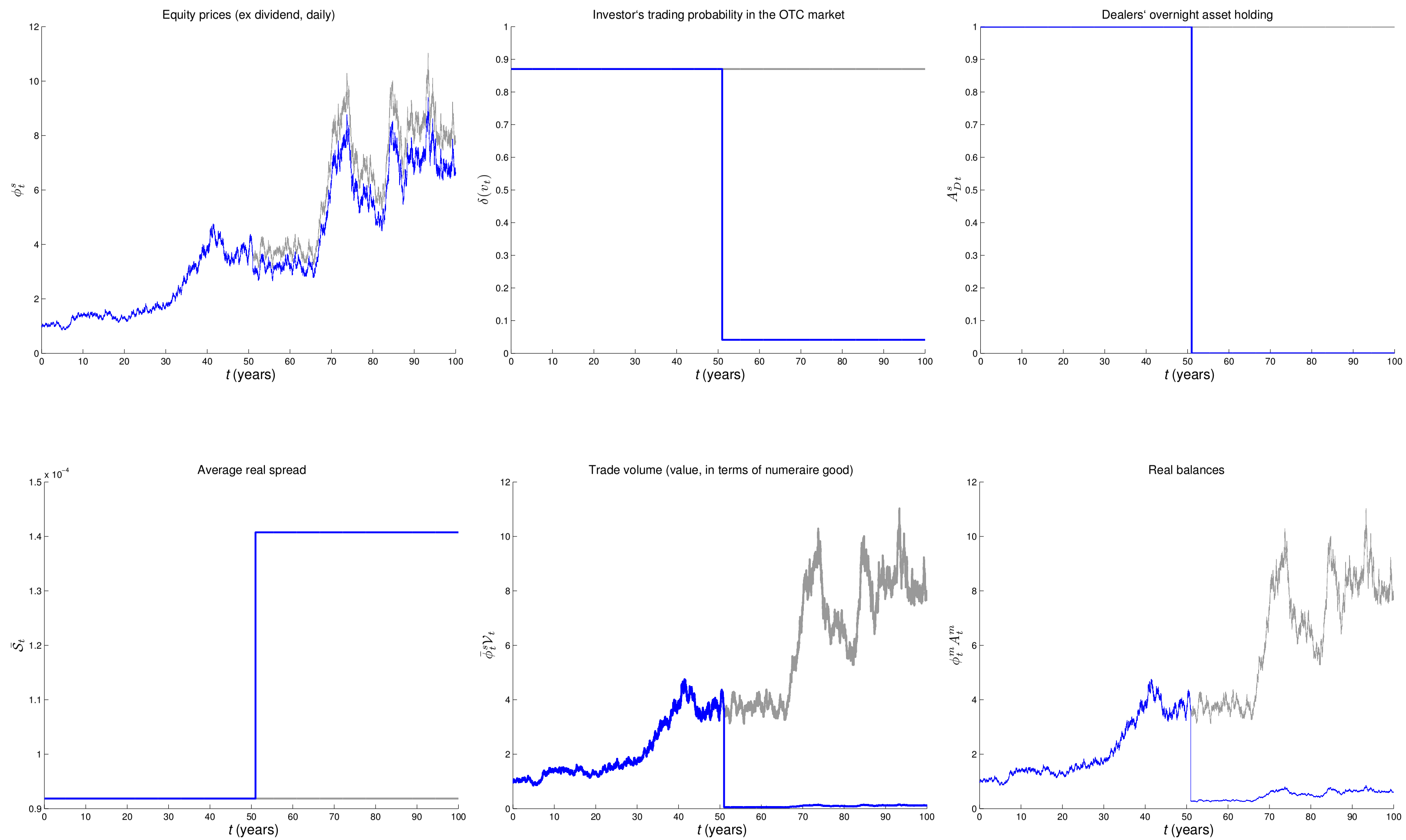

Figure 3: Liquidity crisis